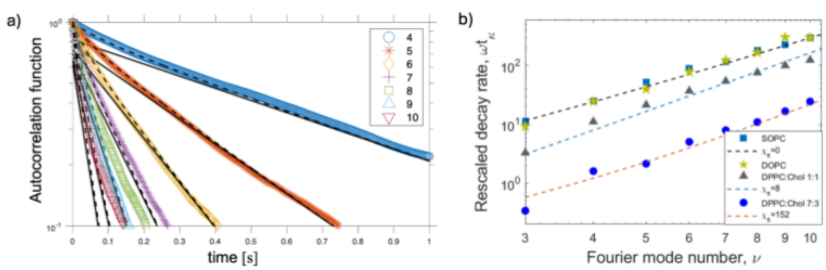
Mathematics of cell membrane-mediated information transmission

Petia Vlahovska and Suri Vaikuntanathan

Biomembranes are an adaptive information-transmitting material utilizing nonequilibrium processes. For example, information transferred between neurons occurs via intermittent voltage pulses-- action potentials --resulting from perturbation and then restoration of the resting potential across the cellular membrane; the transient electric polarization of the membrane is a quintessential non-equilibrium process is physiologically important, yet it is poorly understood. **Our work advances mathematics by developing a new comprehensive model of the action potential that incorporates membrane dynamics far from equilibrium. Our approach generalizes linear irreversible thermodynamics using large deviation theory and stochastic thermodynamics (i.e., a mathematical framework developed to connect thermodynamics and stochastic calculus).** Using this new approach, we use data generated from a non-equilibrium membrane process - in the current work so far, this is data from simulations of non-equilibrium membrane dynamics - and attempt to directly infer a simple low dimensional evolution equation. This new way of deducing minimal laws is rooted in previous theories derived by us.



Mechanical memory: response of biometric membranes to mechanical perturbation decays exponentially with rate dependent on membrane fluidity and the wavelength (curvature) of the perturbation. (a) Time correlations of different wavelength curvature fluctuations of a bilayer membrane. (b) The decay rate decreases with wavenumber (shorter memory) and increases with membrane viscosity, χ, (longer memory).

Using this approach, we have identified low dimensional theoretical laws for membrane dynamics and shapes, and discovered a significant effect of dissipation in membrane curvature fluctuations [1]. We are working to expand on this approach to include electric effects, and apply it to experimental data taken by us for fluctuating electrically-polarized membranes. Ultimately, we hope our efforts lead to new mathematics of the non-equilibrium dynamics of excitable membranes.

[1] H.A. Faizi, R. Granek, and P. M. Vlahovska, "Curvature fluctuations of fluid vesicles reveal hydrodynamic dissipation within the bilayer", Proceedings of the National Academy of Sciences, 121 (44) e2413557121 (2024). This research was supported in part by grants from the NSF (DMS-2235451) and Simons Foundation (MP-TMPS-00005320).