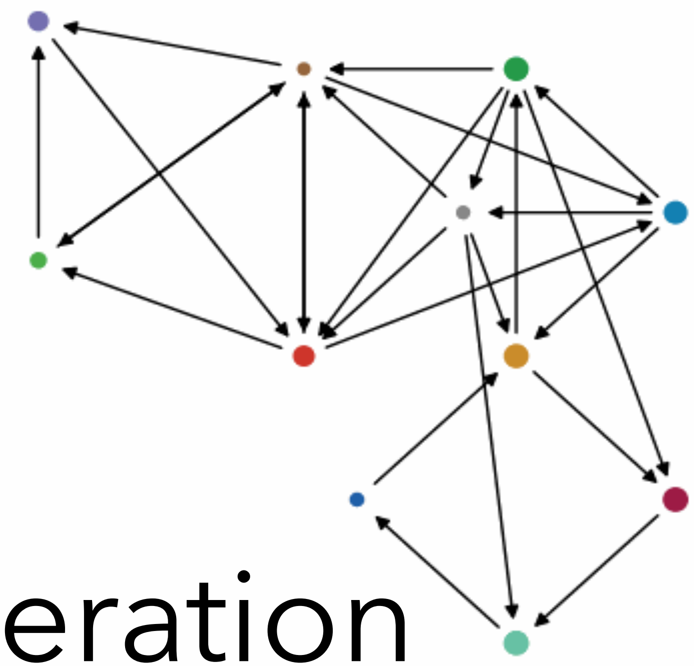
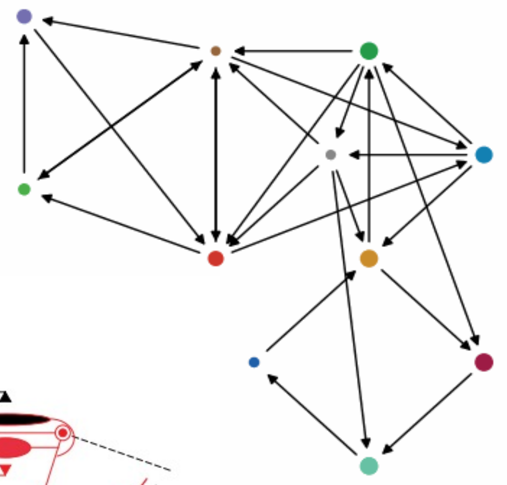
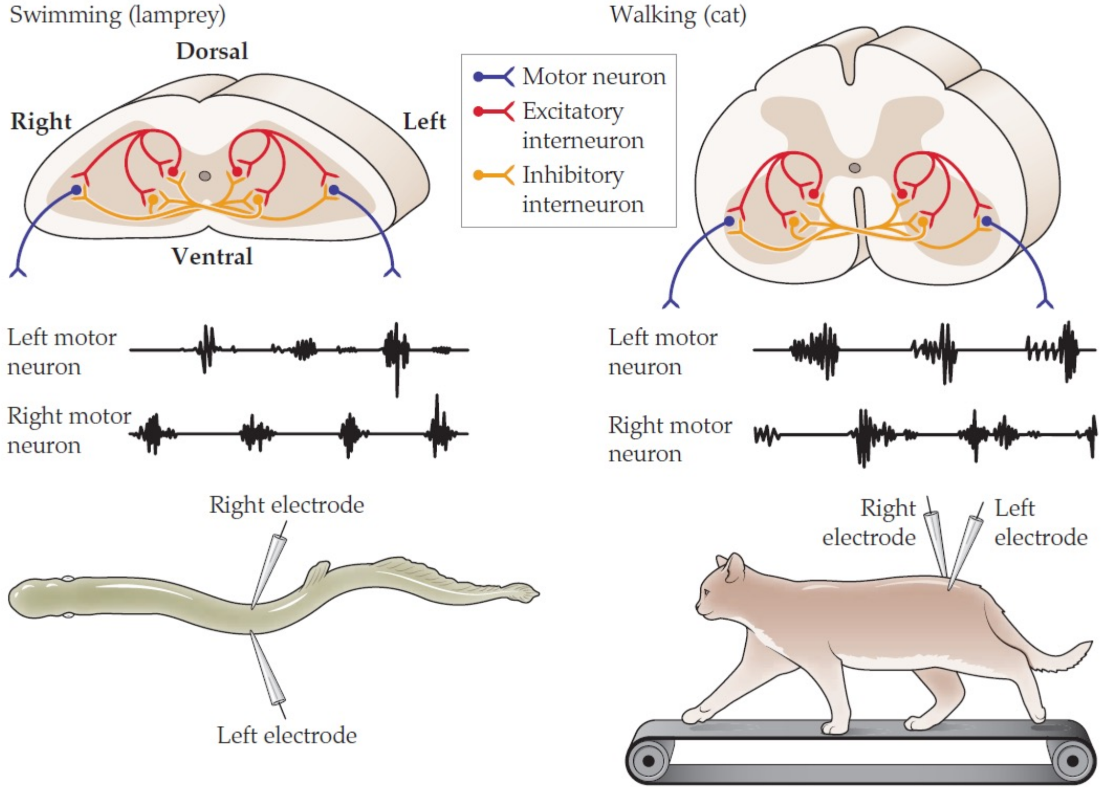


Attractor-based models for sequences and pattern generation in neural circuits

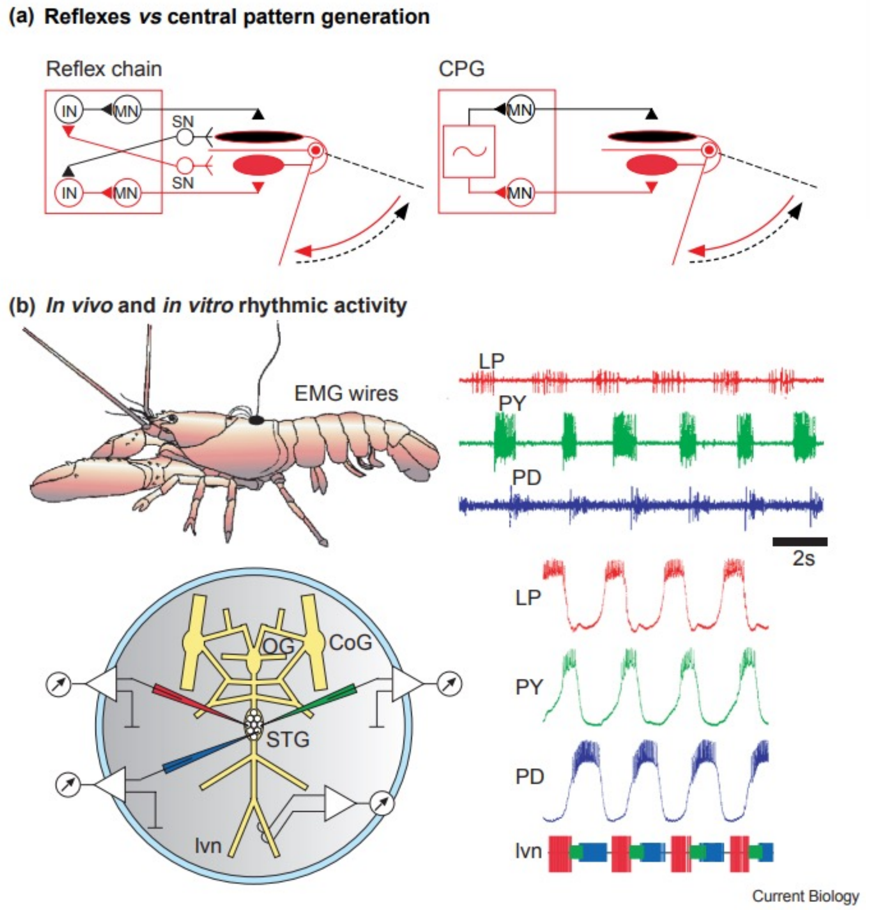
Juliana Londoño Álvarez
Applied Math, Brown University
with Carina Curto and Katie Morrison
August 2025

Oscillations arise in many brain processes



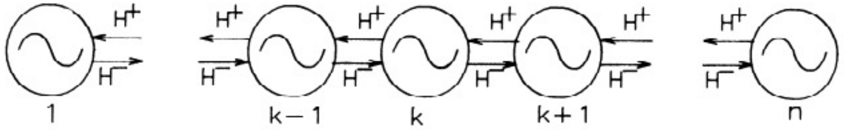


Eagleman & Downar, 2016, Brain and Behavior



Marder & Bucher, 2001, Central Pattern Generators

Long history of coupled oscillator models in rhythm generation



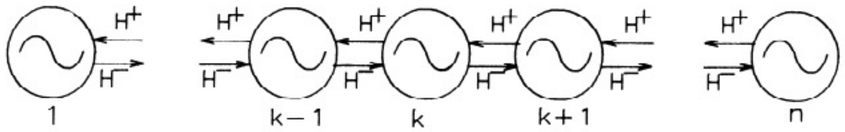
$$d\Theta_k/dt = \omega_k + H^+(\Theta_{k+1} - \Theta_k) + H^-(\Theta_{k-1} - \Theta_k)$$

Williams et al., 1990, *Lamprey CPG*

Theoretical results & locomotion models (~80s to 00s):

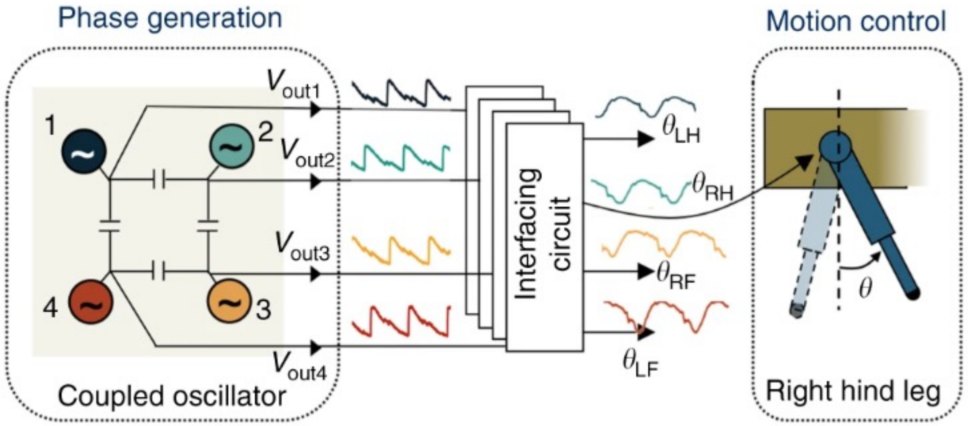
Nancy Kopell, Bard Ermentrout, A.H. Cohen, Martin Golubitsky, Ian Stewart, Pietro-Luciano Buono, J.J. Collins and many more…

Long history of coupled oscillator models in rhythm generation



$$d\Theta_k/dt = \omega_k + H^+(\Theta_{k+1} - \Theta_k) + H^-(\Theta_{k-1} - \Theta_k)$$

Williams et al., 1990, *Lamprey CPG*



Dutta et al., 2019, *Programmable coupled oscillators*

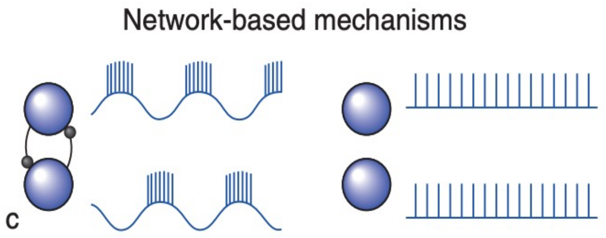
Theoretical results & locomotion models (~80s to 00s):

Nancy Kopell, Bard Ermentrout, A.H. Cohen, Martin Golubitsky, Ian Stewart, Pietro-Luciano Buono, J.J. Collins and many more…

Robotics:

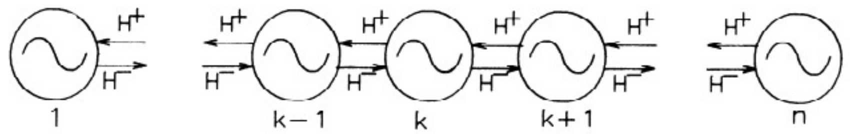
Auke Ijspeert, Suman Datta and many more…

- ☺ lots of theoretical results available
- 😐 periodic solutions might be fine-tuned (not attractors)
- ☹ not all neurons are pacemakers



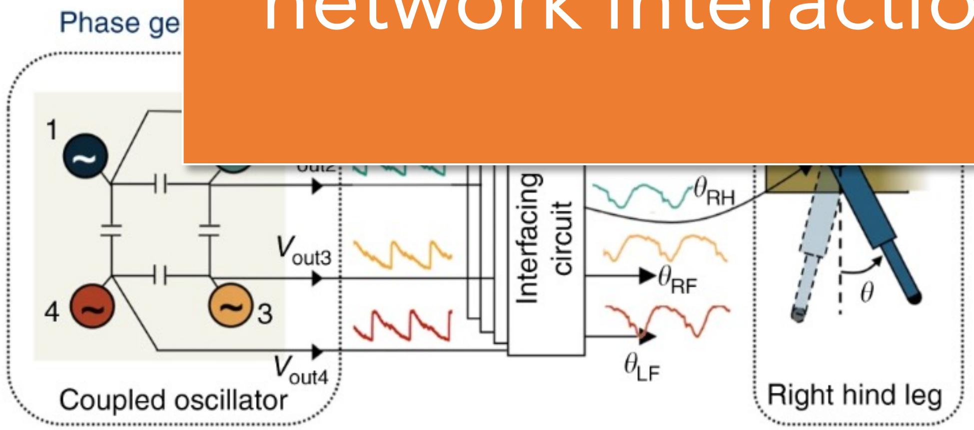
Bucher, 2009, *Central pattern generators*

Long history of coupled oscillator models in rhythm generation



$$d\Theta_k/dt = \omega_k + H^+(\Theta_{k+1} - \Theta_k) + H^-(\Theta_{k-1} - \Theta_k)$$

Williams e



Dutta et al., 2019, *Programmable coupled oscillators*

How do (robust) oscillations emerge from network interactions between non-oscillating units?

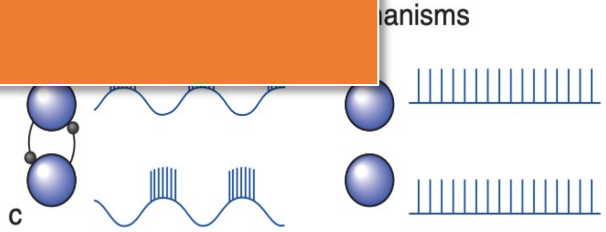
Theoretical results & locomotion models (~80s to 00s):

Robotics:

Auke Ijspeert, Suman Datta and many more…

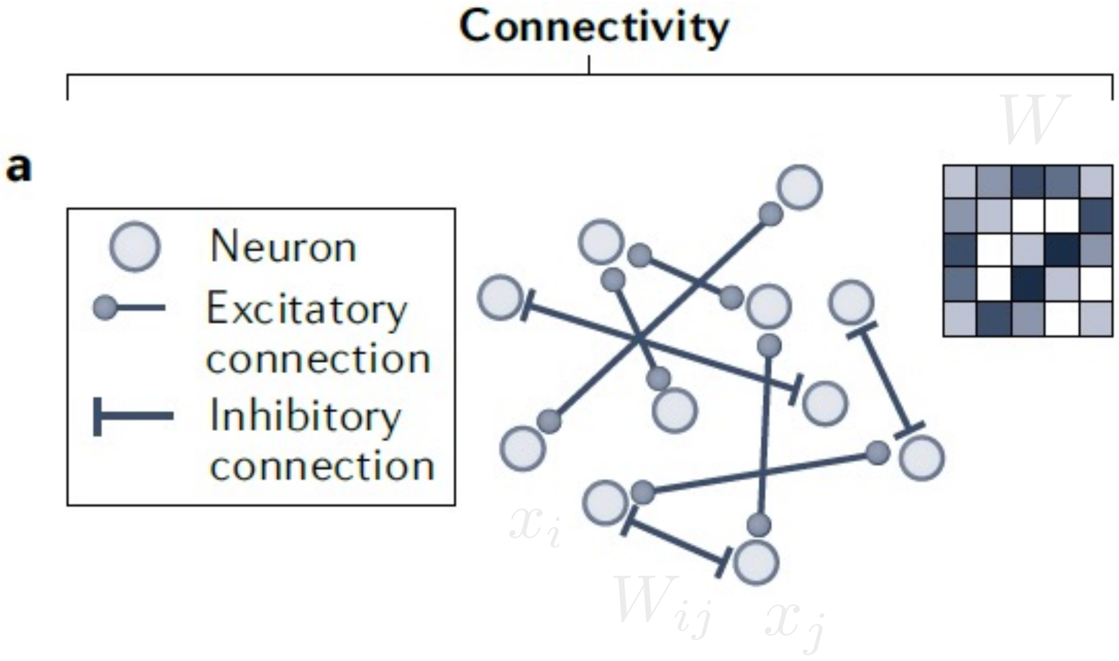
- ☺ lots of theoretical results available
- 😐 periodic solutions might be fine-tuned (not

s are



Bucher, 2009, *Central pattern generators*

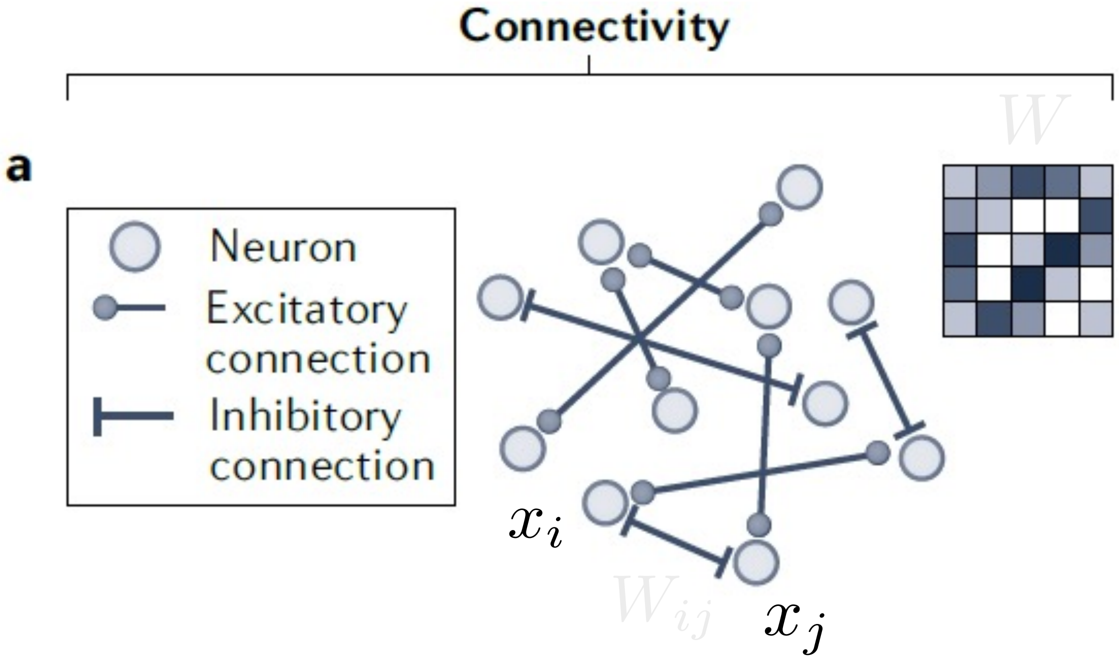
Attractor networks are good at isolating the role of connectivity



$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij} x_j + b_i\right)$$

Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

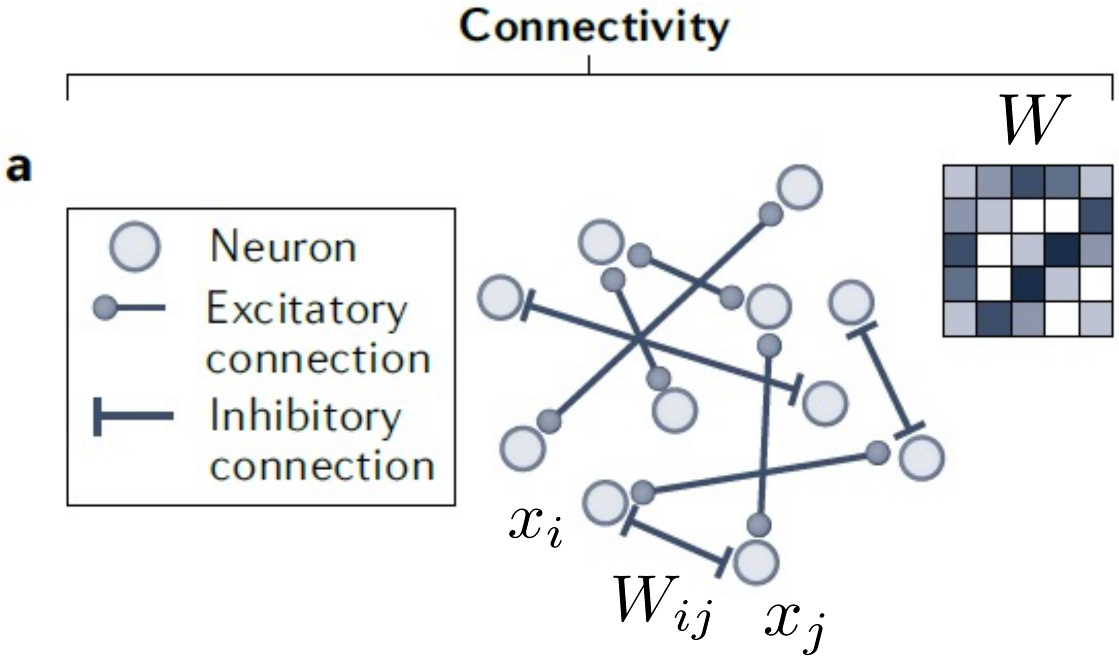
Attractor networks are good at isolating the role of connectivity



$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij}x_j + b_i\right)$$

Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

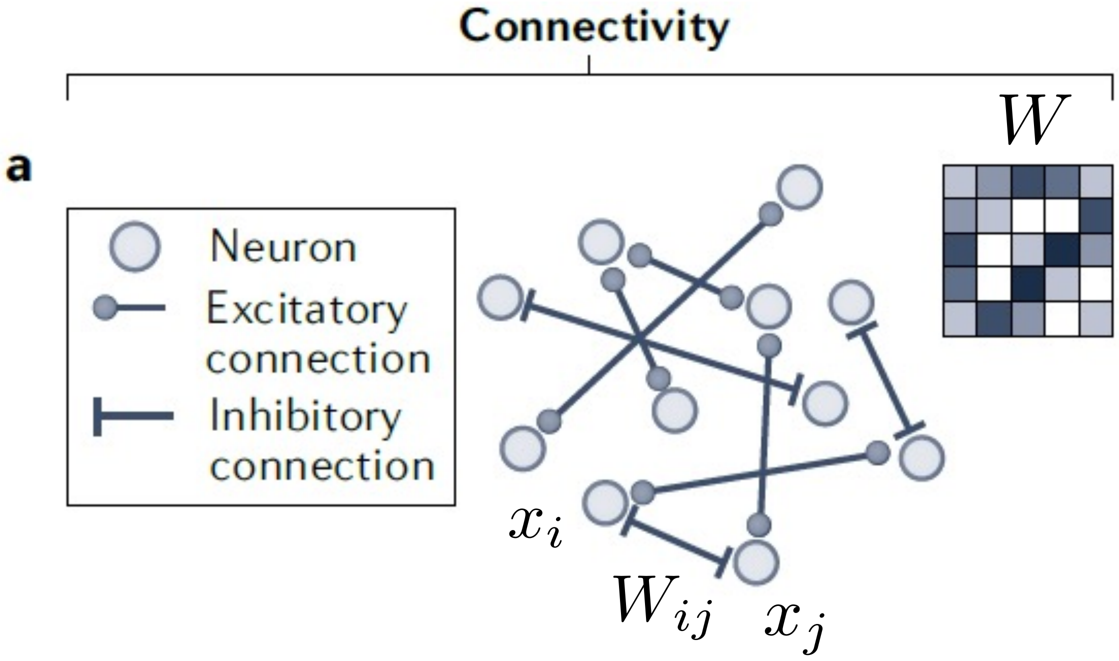
Attractor networks are good at isolating the role of connectivity



$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij} x_j + b_i\right)$$

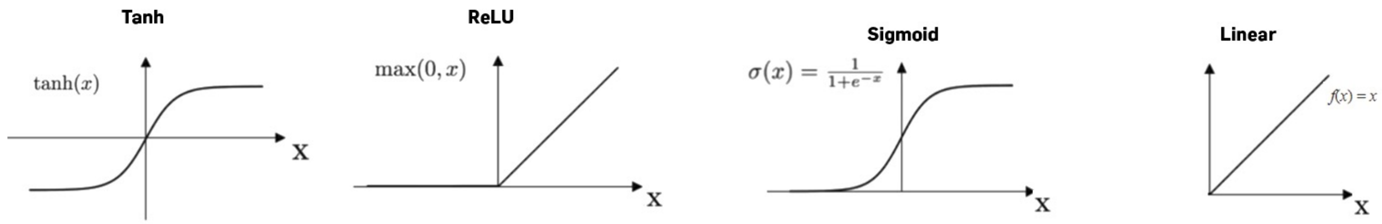
Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

Attractor networks are good at isolating the role of connectivity



$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij} x_j + b_i\right)$$

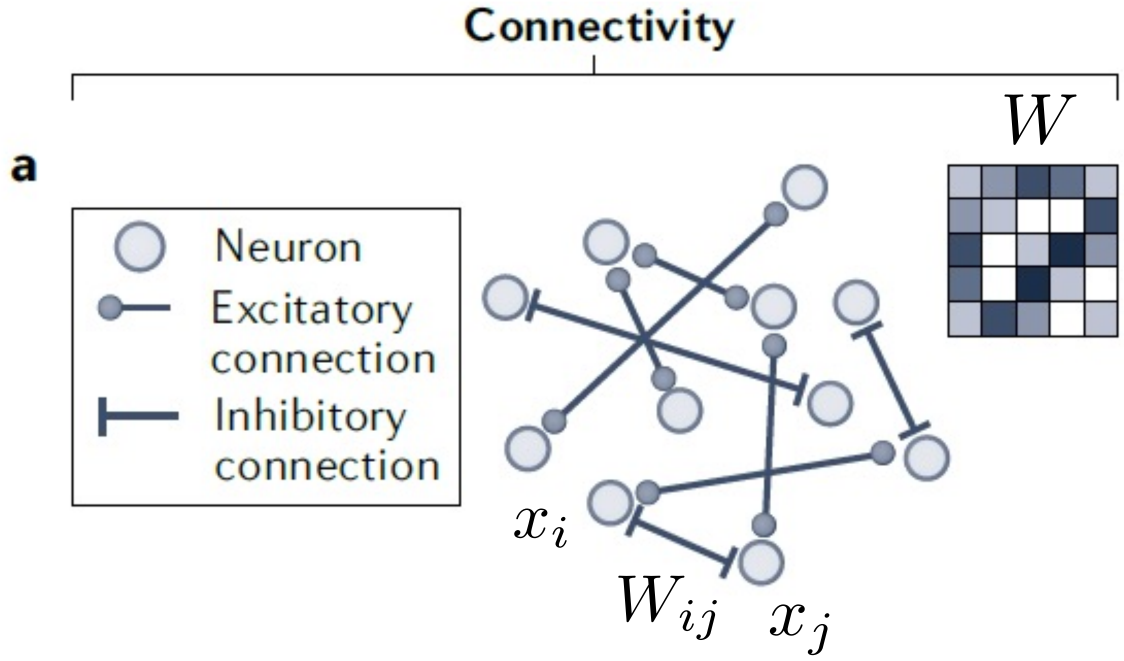
φ



AI Wiki

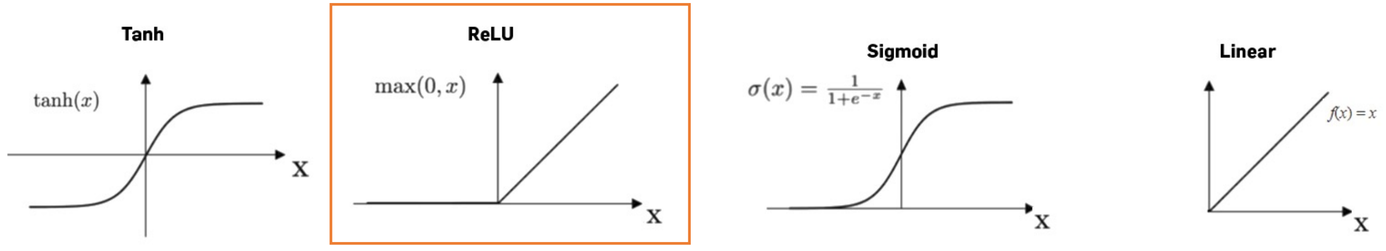
Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

Attractor networks are good at isolating the role of connectivity



$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij}x_j + b_i\right)$$

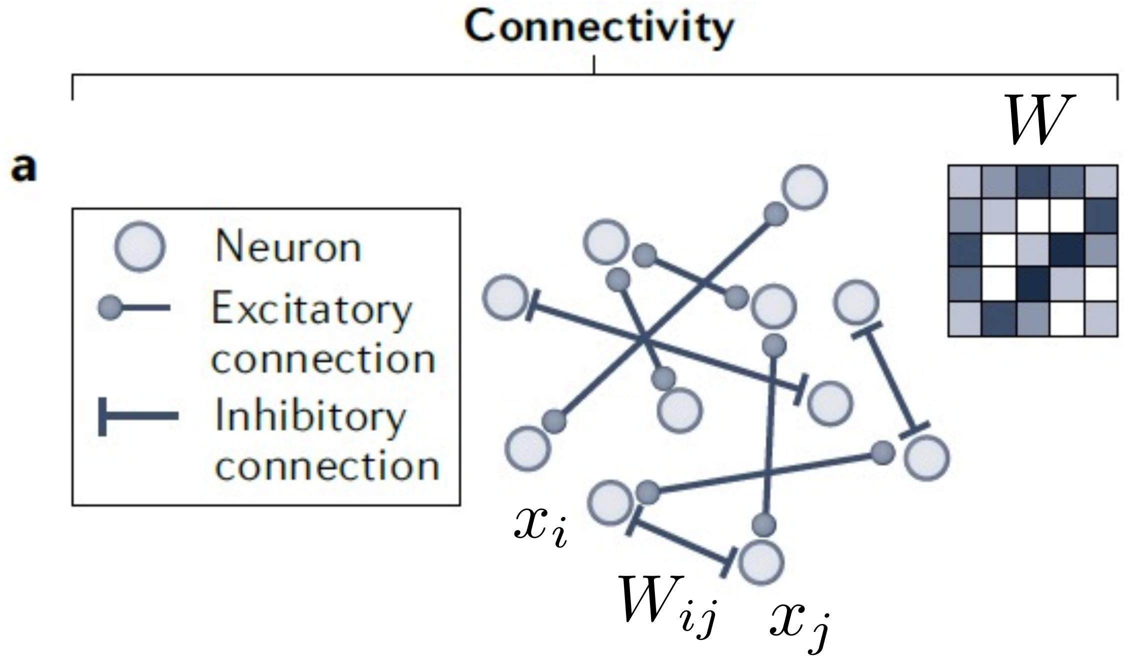
φ



AI Wiki

Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

Attractor networks are good at isolating the role of connectivity

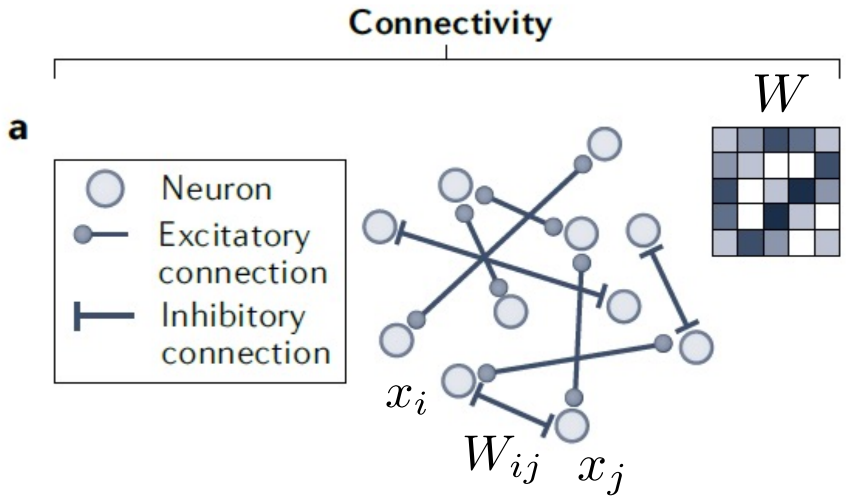


$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij}x_j + b_i\right)$$

Recurrent dynamical network

Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

Attractors are widely used in neuroscience as representations of cognitive processes

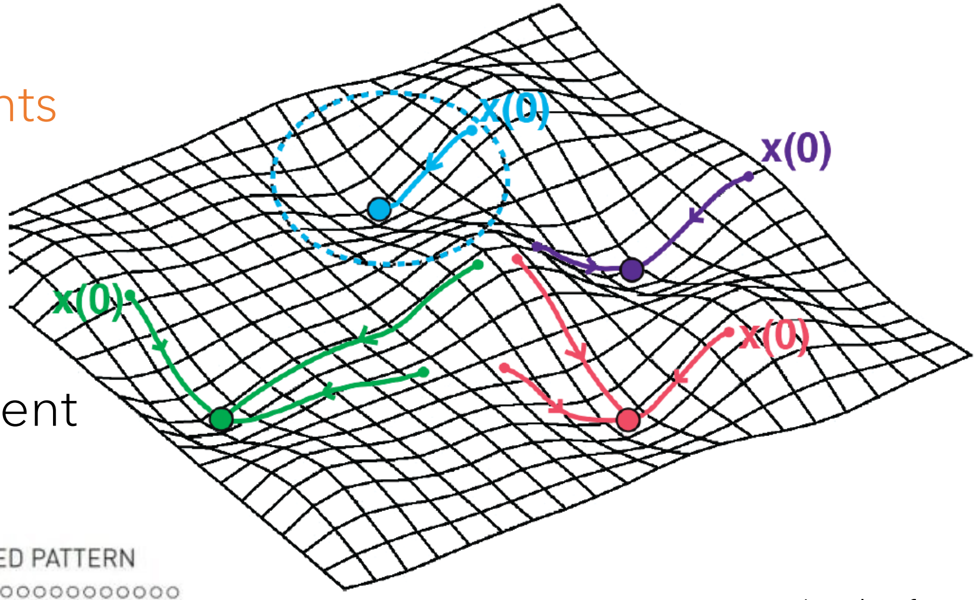


$$\frac{dx_i}{dt} = -x_i + \varphi\left(\sum_{j=1}^{n} W_{ij}x_j + b_i\right)$$

Recurrent dynamical network

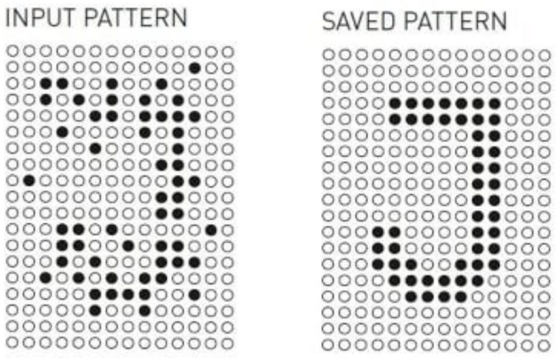
Hopfield networks

Attractors are
stable fixed points



Curto & Morrison, 2023, graph rules for recurrent neural network dynamics

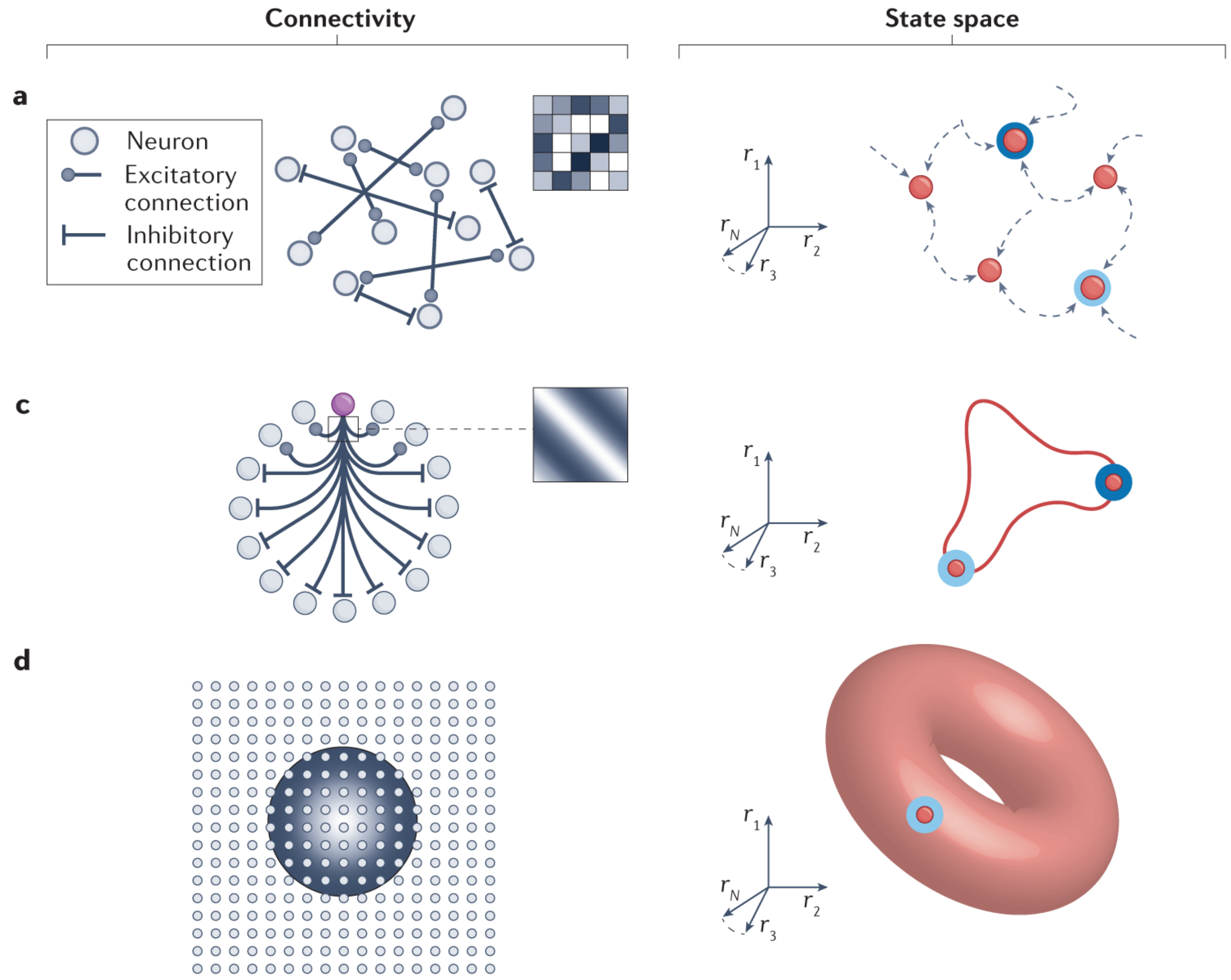
Attractors represent memories



© Johan Jarnestad/The Royal Swedish Academy of Sciences

Adapted from: Khona & Fiete, 2022, *Attractor and integrator networks*

Attractors are widely used in neuroscience as representations of cognitive processes

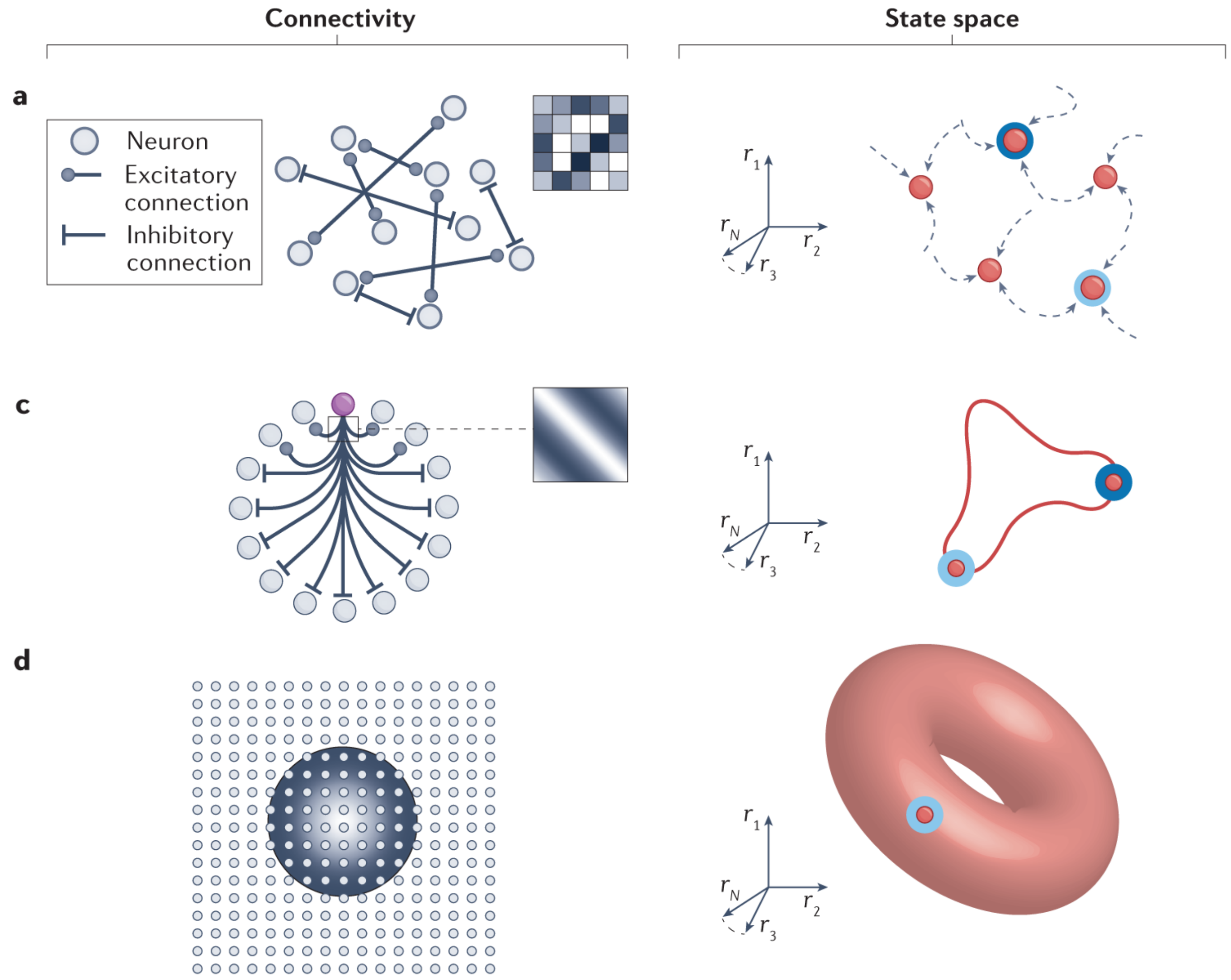


☺ neurons are not pacemakers

$$\frac{dx}{dt} = -x + \varphi\left(Wx + b\right)$$

Khona & Fiete, 2022, *Attractor and integrator networks*

Attractors are widely used in neuroscience as representations of cognitive processes



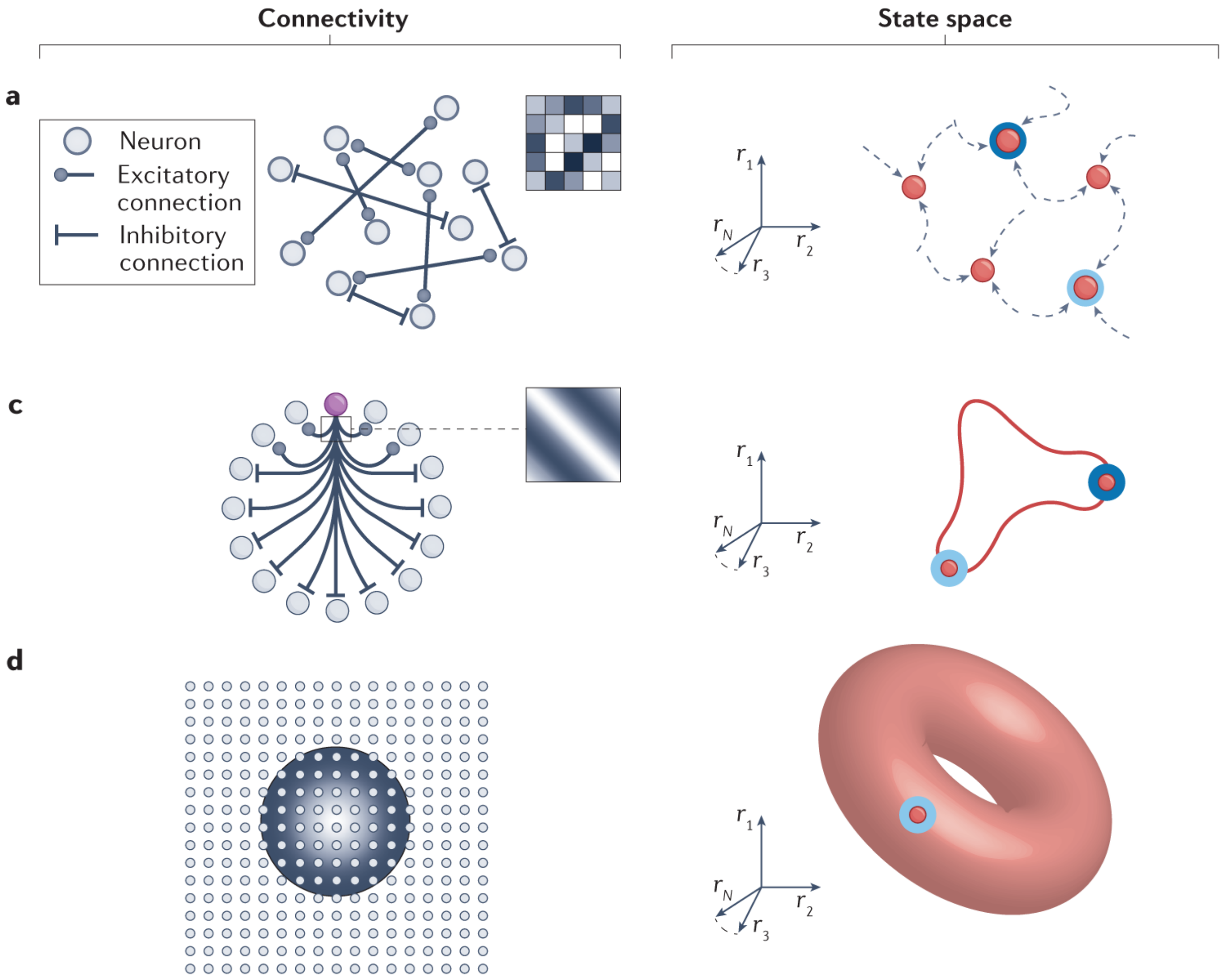
Khona & Fiete, 2022, *Attractor and integrator networks*

☺ neurons are not pacemakers

$$\frac{dx}{dt} = -x + \varphi\left(Wx + b\right)$$

☺ patterns are attractors of the network

Attractors are widely used in neuroscience as representations of cognitive processes



Khona & Fiete, 2022, *Attractor and integrator networks*

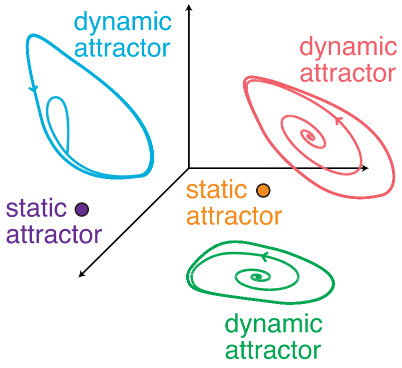
☺ neurons are not pacemakers

$$\frac{dx}{dt} = -x + \varphi\left(Wx + b\right)$$

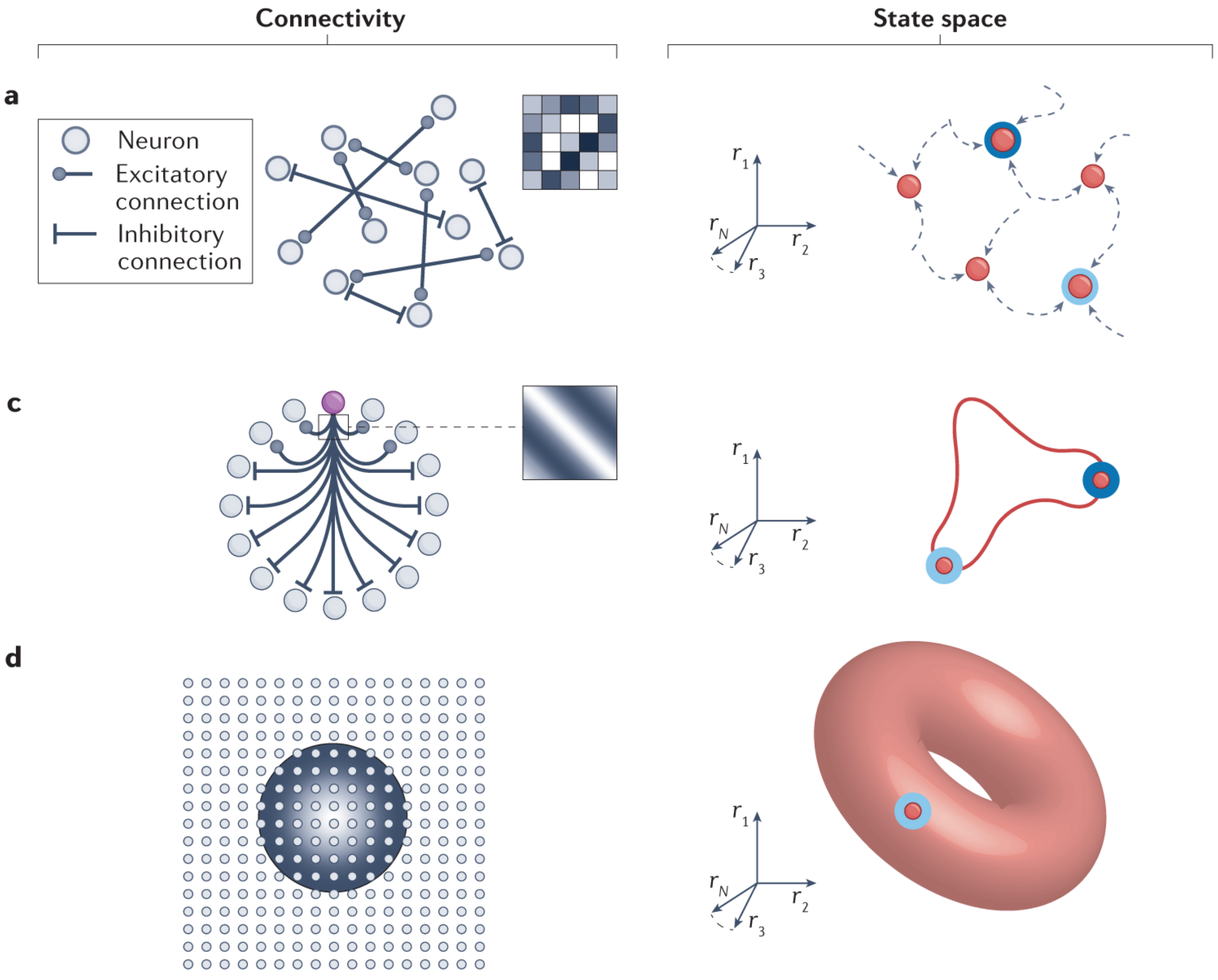
☺ patterns are attractors of the network

☺ lots of theoretical results available…

W = ??



Attractors are widely used in neuroscience as representations of cognitive processes



Khona & Fiete, 2022, *Attractor and integrator networks*

☺ neurons are not pacemakers

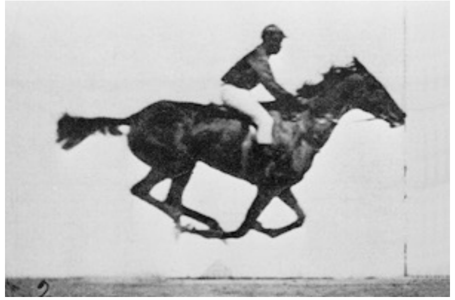
$$\frac{dx}{dt} = -x + \varphi(Wx + b)$$

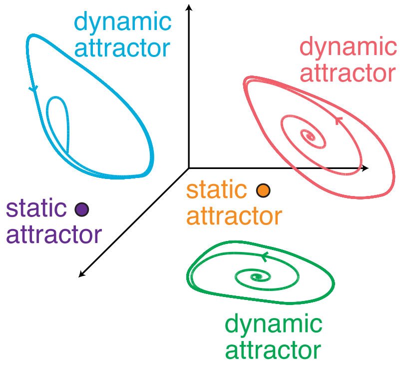
☺ patterns are attractors of the network

☺ lots of theoretical results available…

☹ many of them for *fixed points*

W = ??

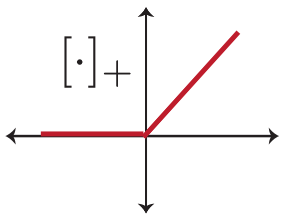




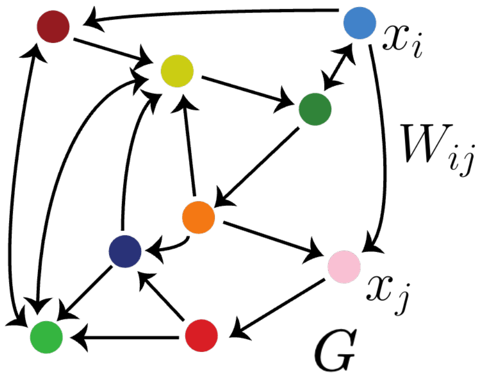
Combinatorial threshold-linear networks (CTLNs)

threshold-linear networks (TLNs):

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij} x_j + b_i\right]_+ \qquad i = 1, \ldots, n$$



whose connectivity matrix is defined by a directed graph:



$$W_{ij} = \begin{cases} 0 & \text{if } i = j \\ -1 + \varepsilon & \text{if } j \to i \text{ in } G \\ -1 - \delta & \text{if } j \not\to i \text{ in } G \end{cases}$$

j weakly inhibits i

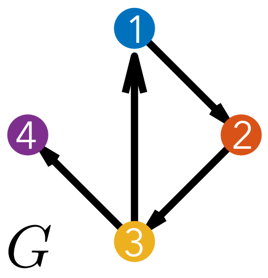
j strongly inhibits i

$b_i = \theta$ Constant input

$\varepsilon, \delta > 0$

$\varepsilon < \dfrac{\delta}{\delta + 1}$

Attractors of (C)TLNs



$$W = \begin{bmatrix} 0 & -1-\delta & -1+\varepsilon & -1-\delta \\ -1+\varepsilon & 0 & -1-\delta & -1+\varepsilon \\ -1-\delta & -1+\varepsilon & 0 & -1-\delta \\ -1-\delta & -1-\delta & -1+\varepsilon & 0 \end{bmatrix} \quad b = \begin{bmatrix} \theta \\ \theta \\ \theta \\ \theta \end{bmatrix}$$

$$\frac{dx}{dt} = -x + [Wx + b]_+$$

$$W_{ij} = \begin{cases} 0 & \text{if } i = j \\ -1+\varepsilon & \text{if } j \to i \text{ in } G \\ -1-\delta & \text{if } j \not\to i \text{ in } G \end{cases}$$

$$\frac{dx^*}{dt} = 0 \qquad x_i^* = \left[\sum_{j=1}^{n} W_{ij} x_j^* + b_i \right]_+$$

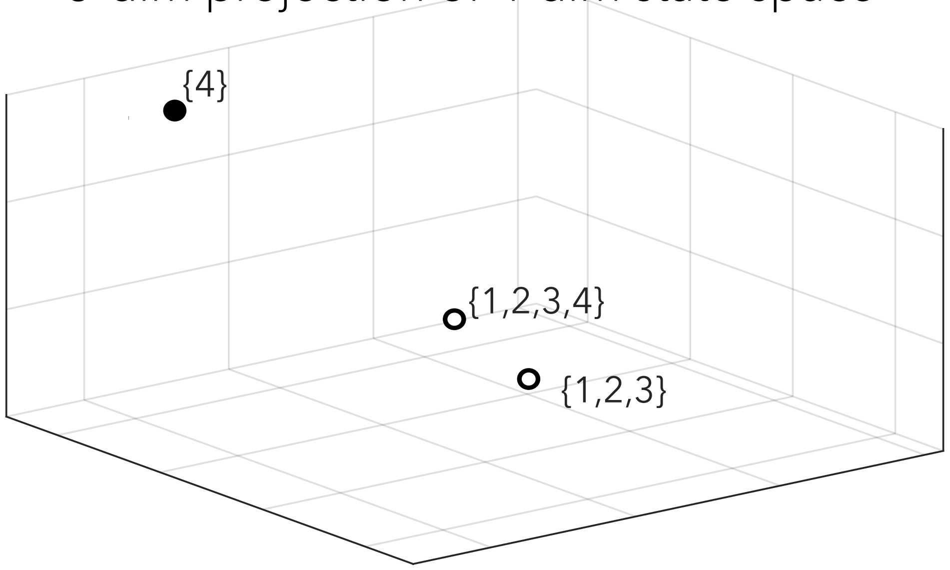
x^* fixed point

$$\sigma = \operatorname{supp}(x^*) = \{ i \mid x_i > 0 \}$$

$\mathrm{FP}(G) = \{ \sigma \subseteq [n] \mid \sigma$ is a fixed point support for $G \}$

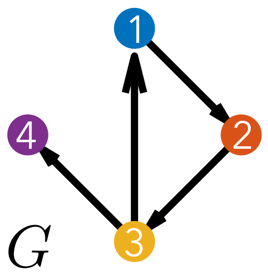
contains stable and unstable fixed points

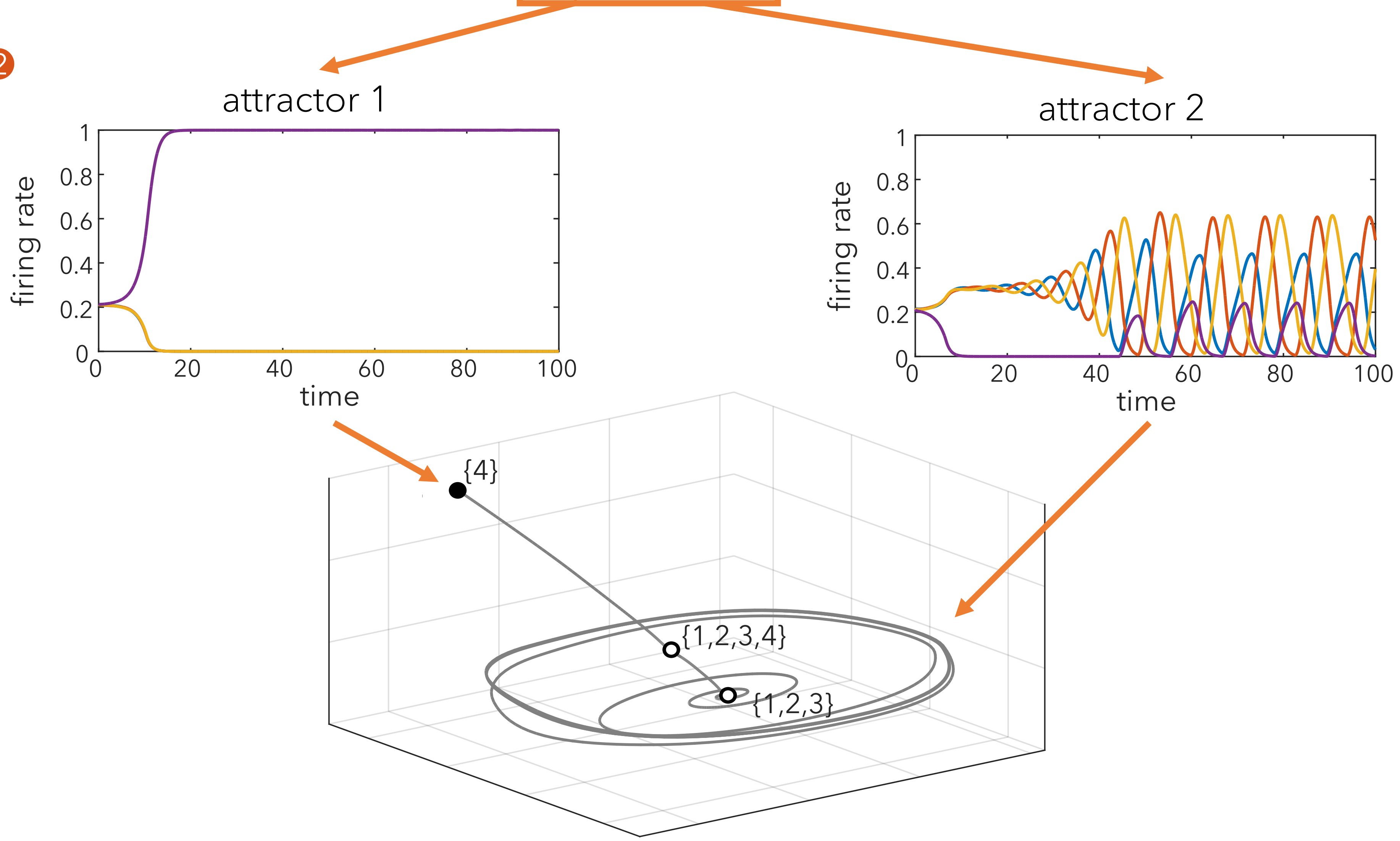
3-dim projection of 4-dim state space



Heuristic: minimal fixed points *correspond* to attractors

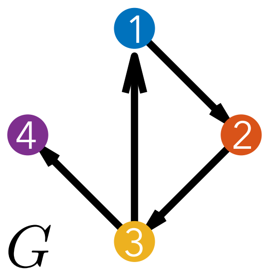
$$\mathrm{FP}(G) = \{\{4\}, \{1,2,3\}, \{1,2,3,4\}\}$$

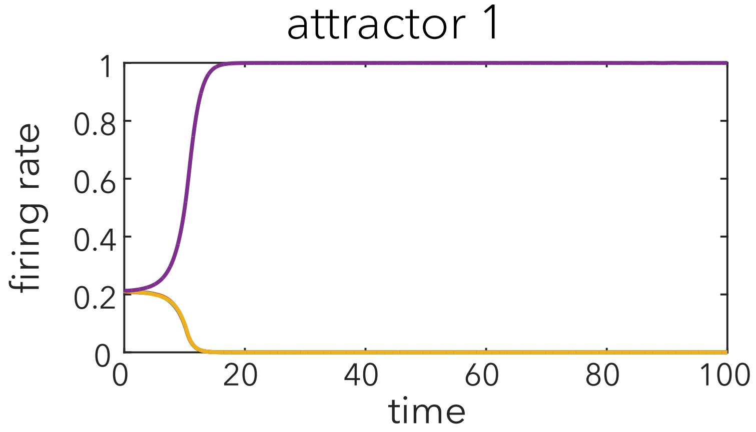


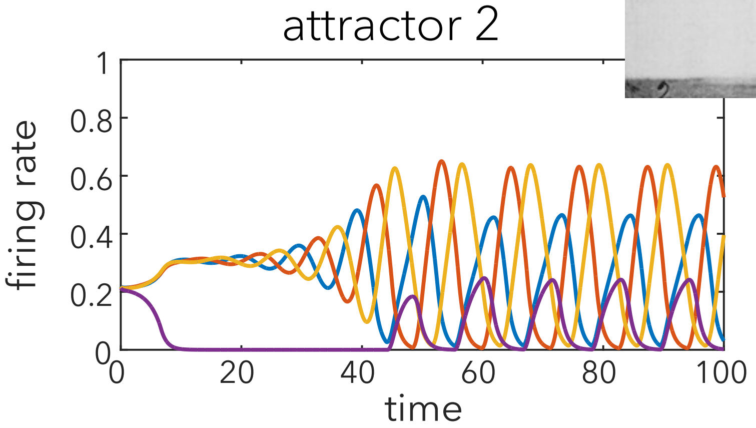


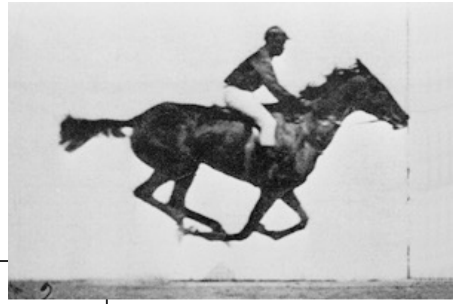
Heuristic: minimal fixed points *correspond* to attractors

$$\mathrm{FP}(G) = \{\{4\}, \{1,2,3\}, \{1,2,3,4\}\}$$





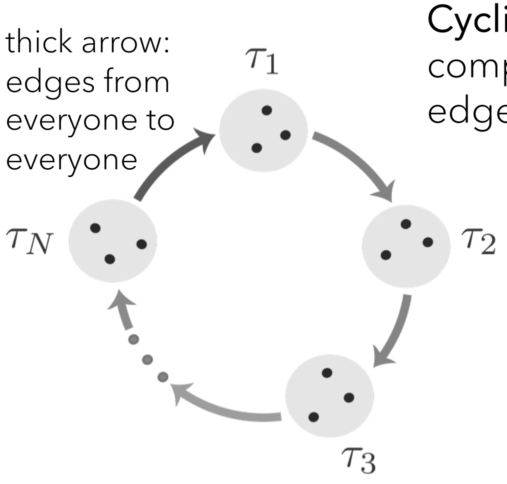




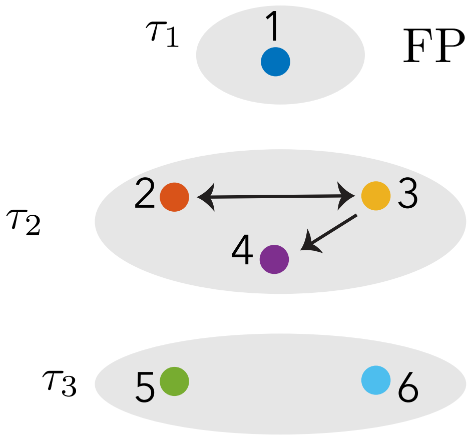
Takeaways:

1. a single network can have many different attractors (which can be a mix of static and dynamic), accessible via different initial conditions
2. FP(G) also gives information about the initial conditions needed to converge to its corresponding attractor
3. it seems easy to have dynamic attractors, but what about more complex patterns of oscillations?

For more complex patterns of oscillations: cyclic unions



Cyclic union: edges forward from every node in previous component to every node in next component, and no other edges between components.



$$\mathrm{FP}(G|_{\tau_1}) = \{\{\mathbf{1}\}\}$$

$$\mathrm{FP}(G|_{\tau_2}) = \{\{\mathbf{4}\}, \{\mathbf{2}, \mathbf{3}\}, \{2, 3, 4\}\}$$

$$\mathrm{FP}(G|_{\tau_3}) = \{\{\mathbf{5}\}, \{\mathbf{6}\}, \{5, 6\}\}$$

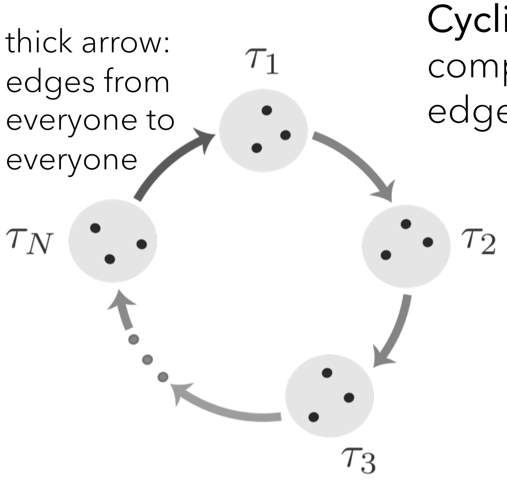
Theorem: for all $i \in [N]$

$$\sigma \in \mathrm{FP}(G) \Leftrightarrow \sigma \cap \tau_i \in \mathrm{FP}(G|_{\tau_i})$$

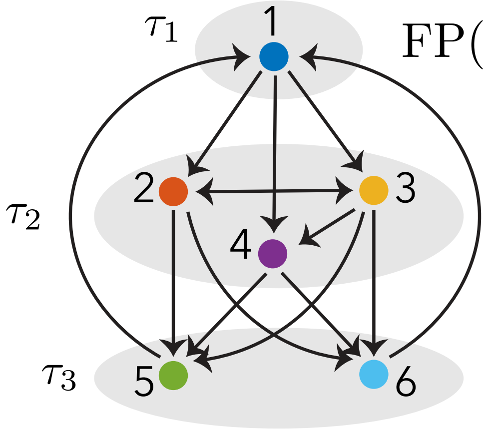
$$|\,\mathrm{FP}(G)| = \prod_{i=1}^{N} \left|\mathrm{FP}\left(G|_{\tau_i}\right)\right|$$

Parmelee, C., **Londono Alvarez, J.**, Curto, C., & Morrison, K. (2022). Sequential attractors in combinatorial threshold-linear networks. *SIAM Journal on Applied Dynamical Systems*, 21(2), 1597-1630.

For more complex patterns of oscillations: cyclic unions



Cyclic union: edges forward from every node in previous component to every node in next component, and no other edges between components.



$\mathrm{FP}(G|_{\tau_1}) = \{\{\mathbf{1}\}\}$ $\quad |\mathrm{FP}(G|_{\tau_1})| = 1$

$\mathrm{FP}(G|_{\tau_2}) = \{\{\mathbf{4}\}, \{\mathbf{2}, \mathbf{3}\}, \{2, 3, 4\}\}$ $\quad |\mathrm{FP}(G|_{\tau_2})| = 3$

$\mathrm{FP}(G|_{\tau_3}) = \{\{\mathbf{5}\}, \{\mathbf{6}\}, \{5, 6\}\}$ $\quad |\mathrm{FP}(G|_{\tau_3})| = 3$

Theorem: for all $i \in [N]$

$$\sigma \in \mathrm{FP}(G) \Leftrightarrow \sigma \cap \tau_i \in \mathrm{FP}(G|_{\tau_i})$$

$$|\mathrm{FP}(G)| = \prod_{i=1}^{N} |\mathrm{FP}(G|_{\tau_i})|$$

theorem says:

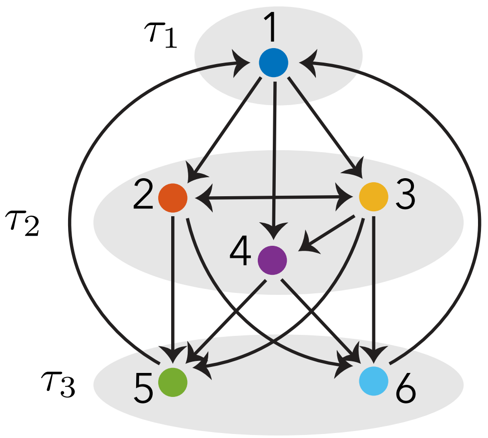
$$\mathrm{FP}(G) = \{\{\mathbf{1}, \mathbf{4}, \mathbf{5}\}, \{\mathbf{1}, \mathbf{4}, \mathbf{6}\}, \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{5}\}, \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{6}\}, \{1, 4, 5, 6\}, \{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}, \{1, 2, 3, 5, 6\}, \{1, 2, 3, 4, 5, 6\}\}$$

$|\mathrm{FP}(G)| = 1 * 3 * 3 = 9$

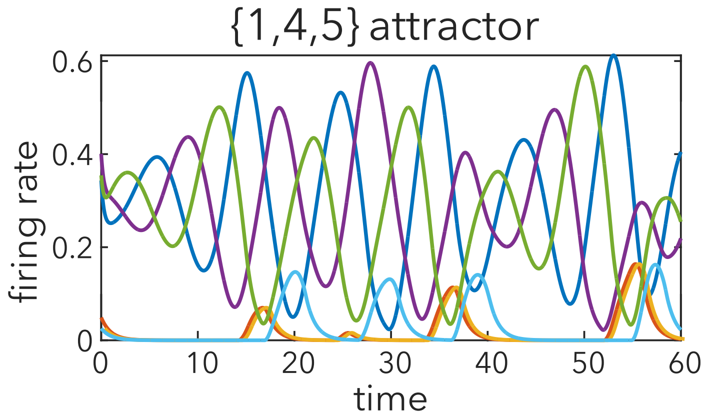
4 of them minimal

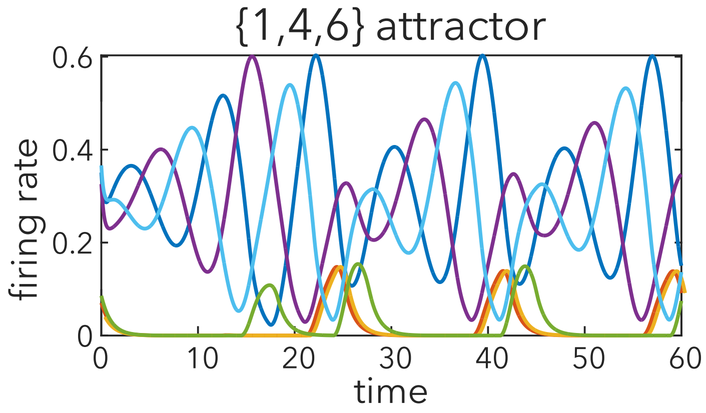
Parmelee, C., **Londono Alvarez, J.**, Curto, C., & Morrison, K. (2022). Sequential attractors in combinatorial threshold-linear networks. *SIAM Journal on Applied Dynamical Systems*, 21(2), 1597-1630.

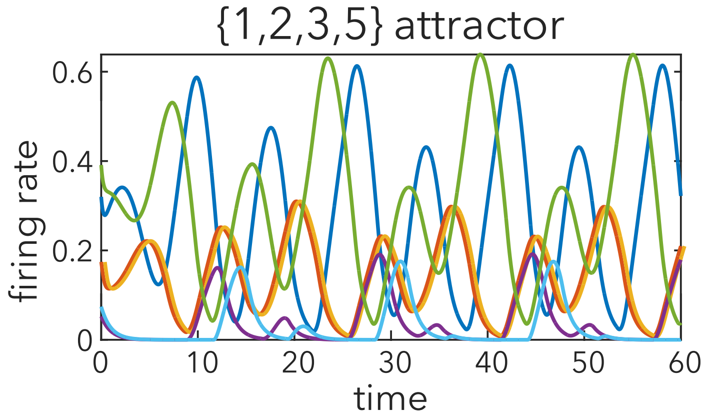
For more complex patterns of oscillations: cyclic unions

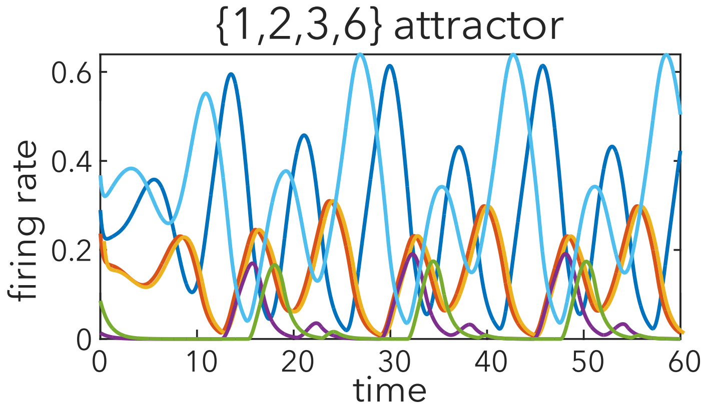


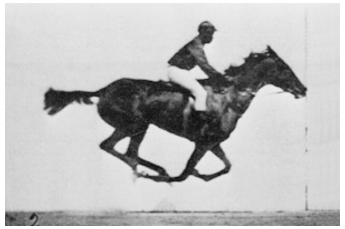
$$\mathrm{FP}(G) = \{\{\mathbf{1},\mathbf{4},\mathbf{5}\}, \{\mathbf{1},\mathbf{4},\mathbf{6}\}, \{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{5}\}, \{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{6}\}, \{1,4,5,6\}, \{1,2,3,4,5\}, \{1,2,3,4,6\}, \{1,2,3,5,6\}, \{1,2,3,4,5,6\}\}$$





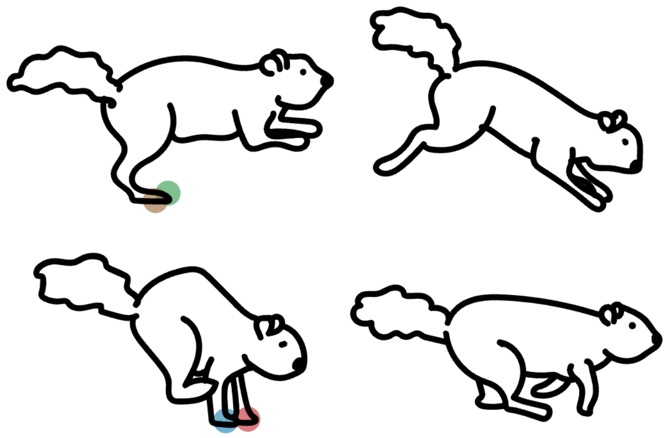


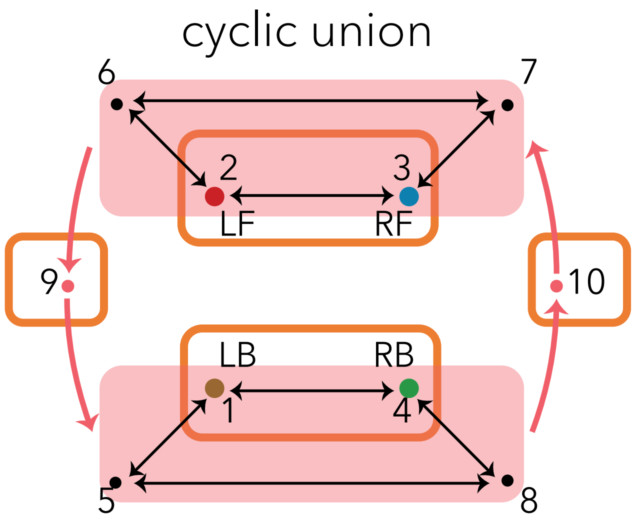




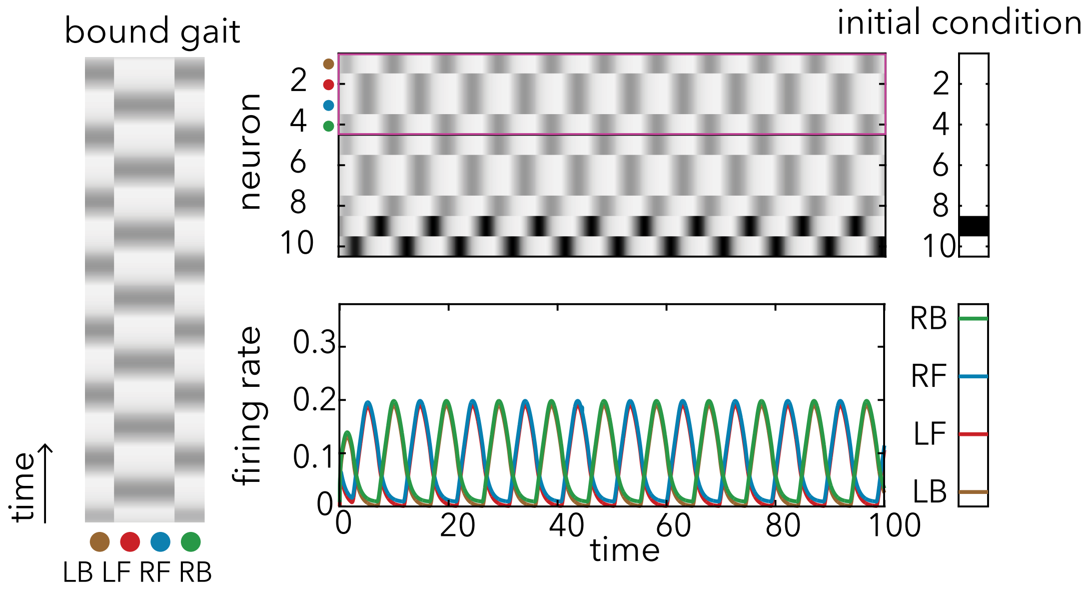
Cyclic union example: bound gait

Front legs are synchronized, back legs are synchronized, half a period apart.





$$|\mathrm{FP}(G)| = 9 * 1 * 9 * 1 = 81$$



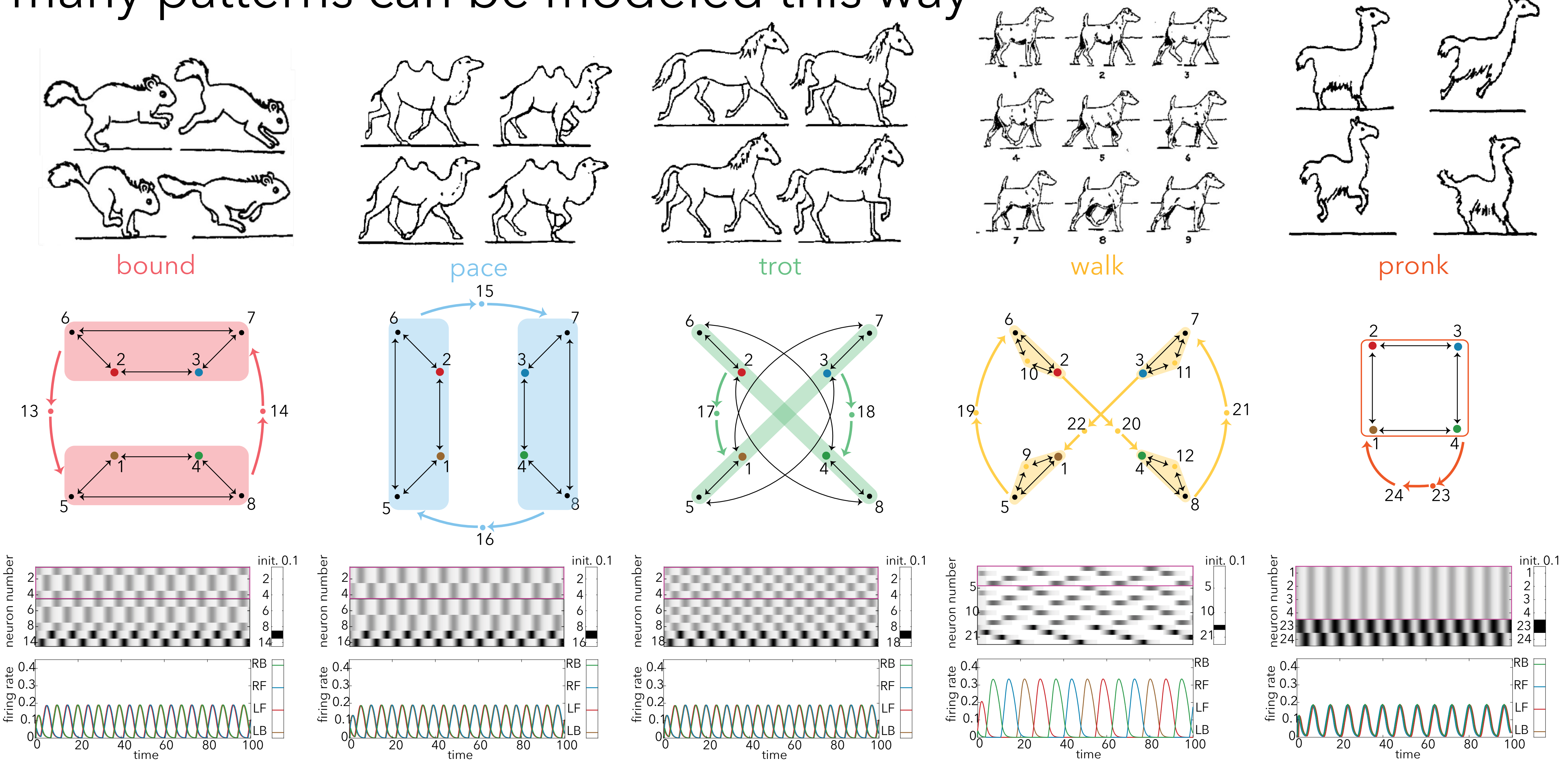
Node-leg assignment

$$\text{minimal supports} = \{\boxed{\{1,2,3,4,9,10\}}, \{1,2,3,5,9,10\}, \{1,2,4,6,9,10\}, \{1,2,5,6,9,10\}, \{1,3,4,7,9,10\}, \{1,3,5,7,9,10\}, \{1,4,6,7,9,10\}, \{1,5,6,7,9,10\}, \{2,3,4,8,9,10\}, \{2,3,5,8,9,10\}, \{2,4,6,8,9,10\}, \{2,5,6,8,9,10\}, \{3,4,7,8,9,10\}, \{3,5,7,8,9,10\}, \{4,6,7,8,9,10\}, \{5,6,7,8,9,10\}\}$$

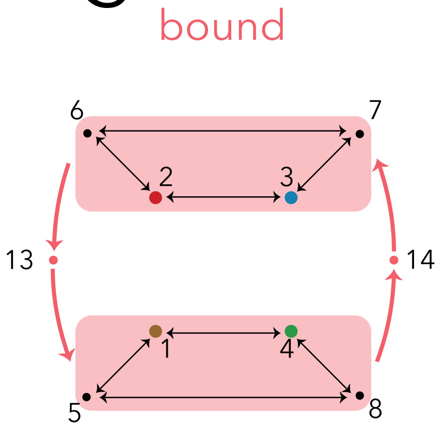
Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.

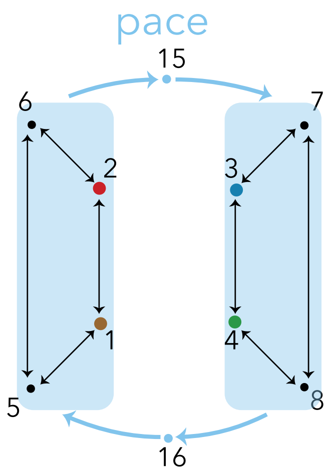
Animal drawings: van der Weele, J. P., & Banning, E. J. (2001). *American Journal of Physics*.

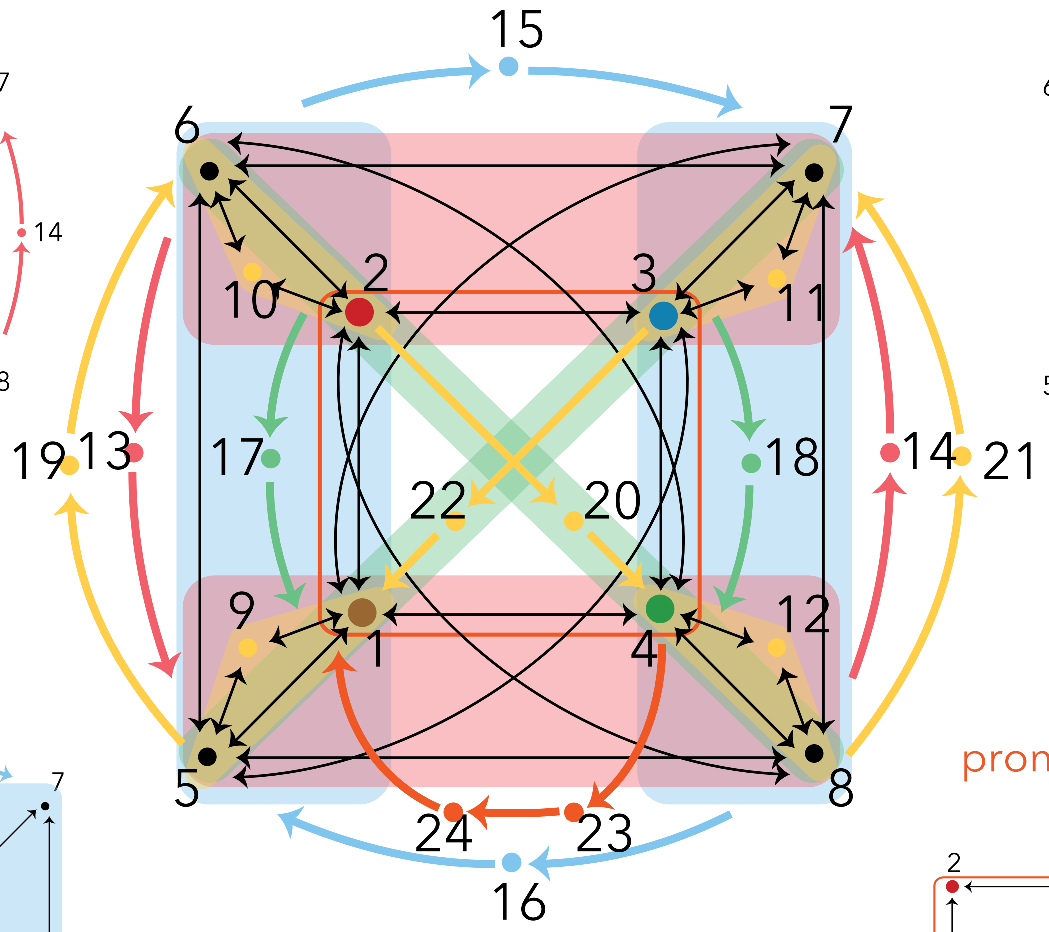
many patterns can be modeled this way

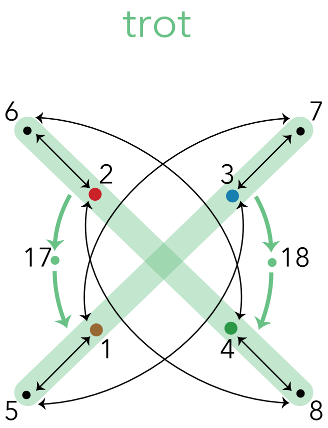


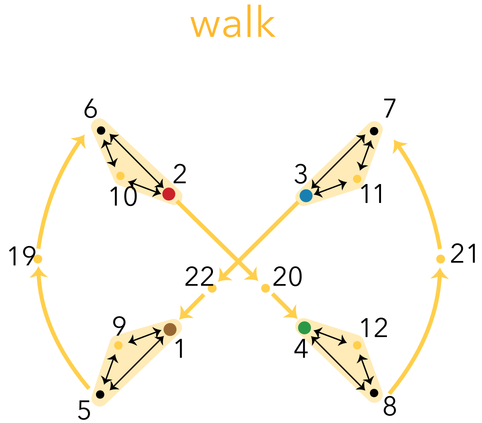
remarkable (unexplained) robustness of construction: all gaits can be merged into a single network

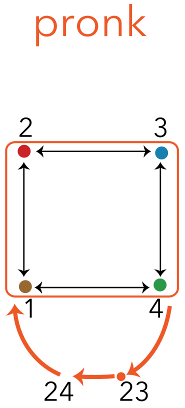






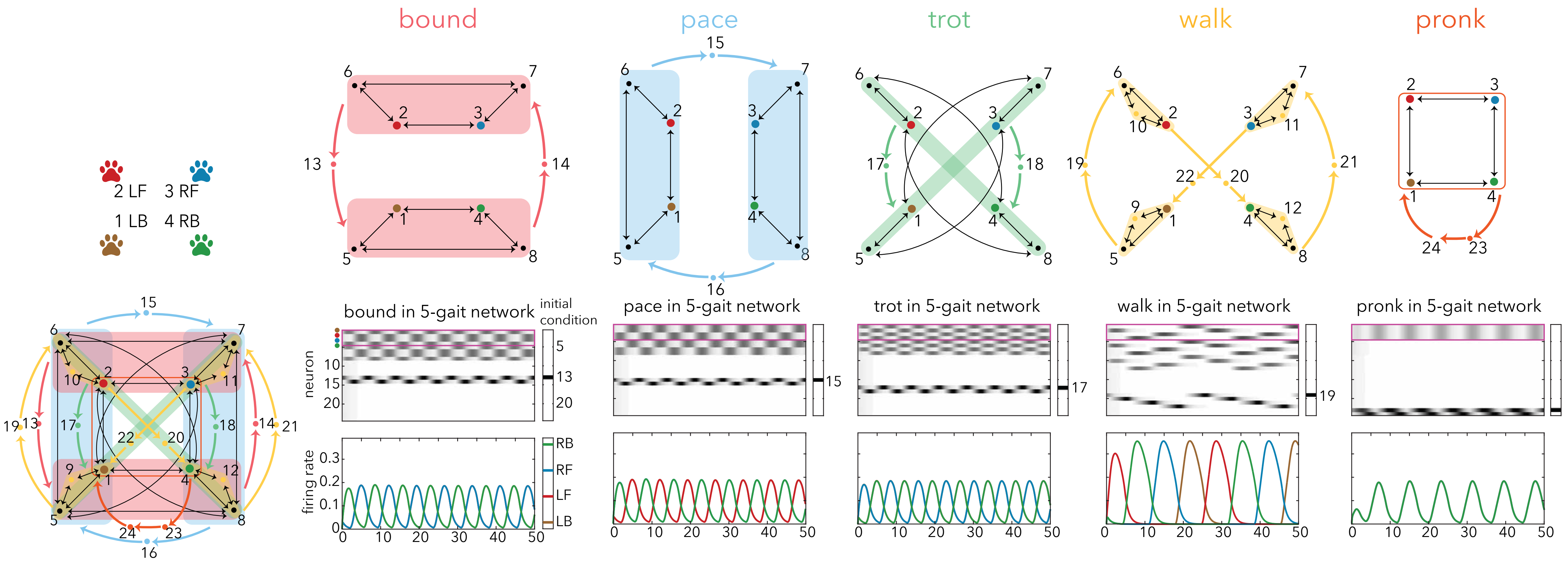






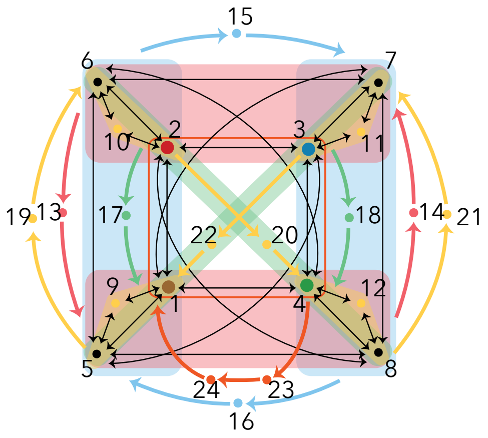
Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.

all attractors survive the merge

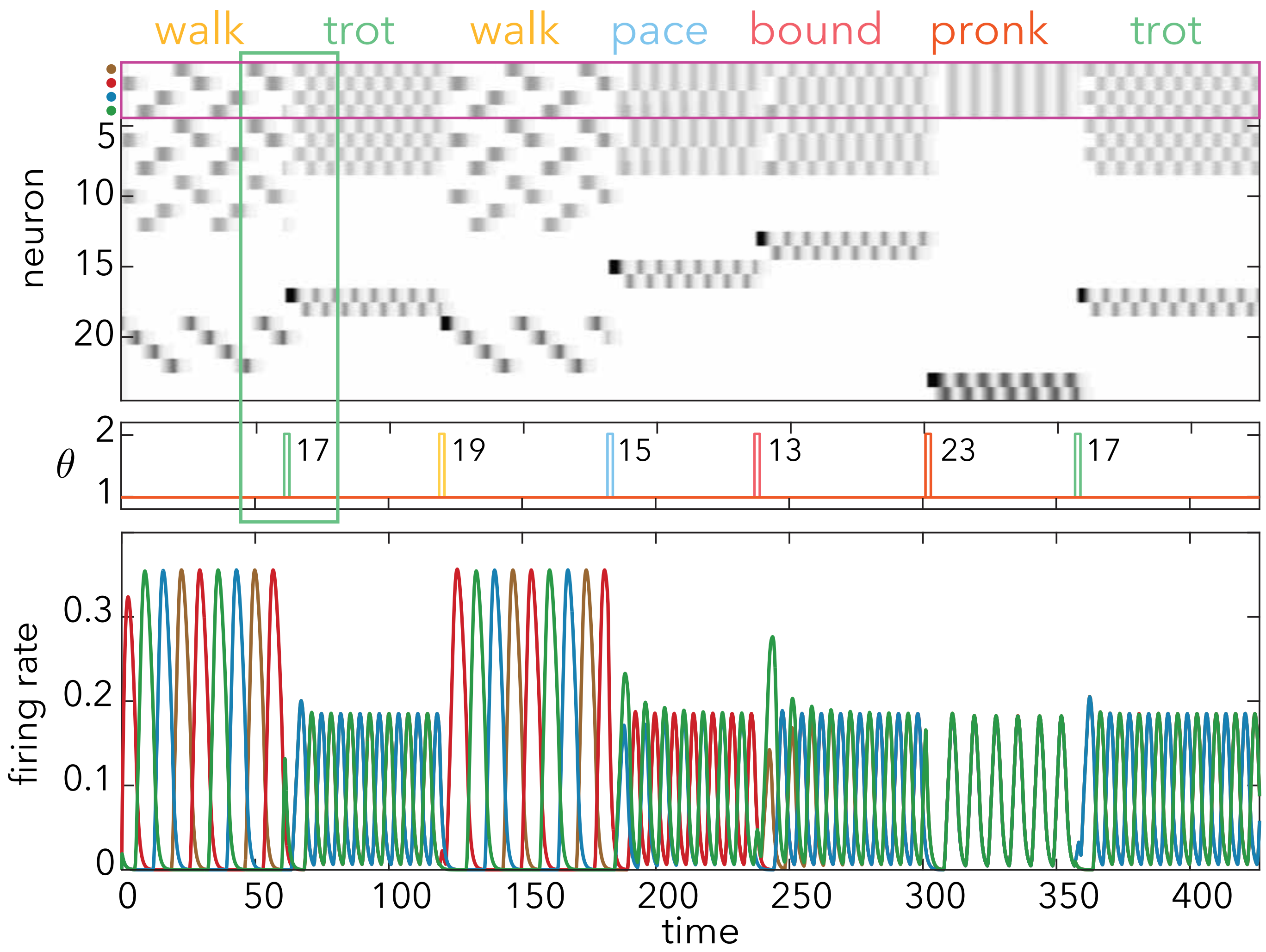


Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.

network can easily transition attractors via pulses sent to auxiliary neurons



Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.



$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

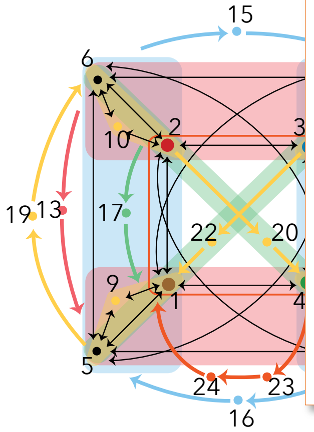
network can easily transition attractors via pulses sent to auxiliary neurons

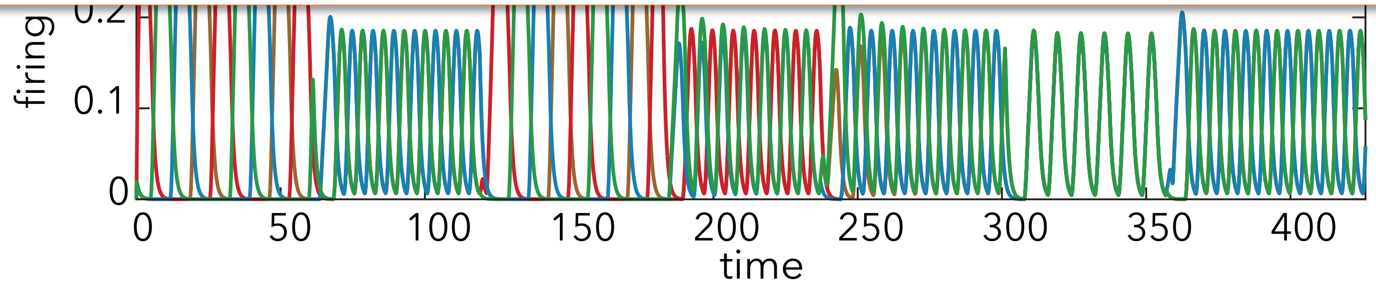
walk trot walk pace bound pronk trot

This network:

1. has five different coexistent attractors, all engaging the same four limb neurons in different patterns
2. has all these attractors accessible without changing parameters (accessed via different initial conditions)
3. allows easy transitions between attractors by stimulating single neurons
4. has attractors that look nearly identical as the individually constructed gaits

What made these attractors so robust that they don't interfere with each other when "glued" together?



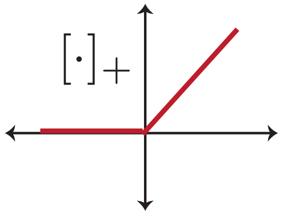


Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.

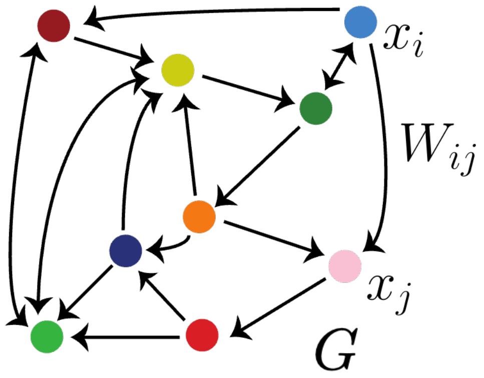
combinatorial threshold-linear networks (CTLNs)

threshold-linear networks:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+ \qquad i = 1, \ldots, n$$



whose connectivity matrix is defined by a directed graph:



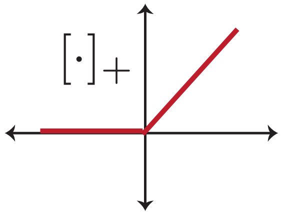
$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1 + \varepsilon & \text{if } j \to i \text{ in } G, \\ -1 - \delta & \text{if } j \not\to i \text{ in } G, \end{cases}$$

$$b_i = \theta$$

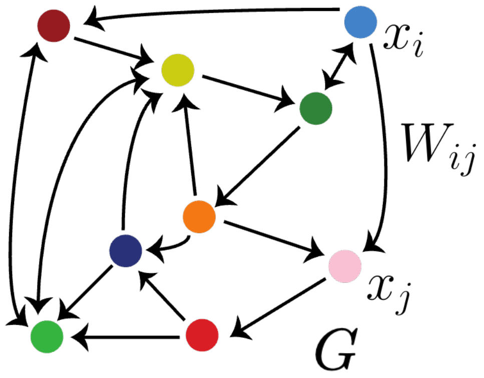
combinatorial threshold-linear networks (CTLNs)

threshold-linear networks:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+ \qquad i = 1, \ldots, n$$

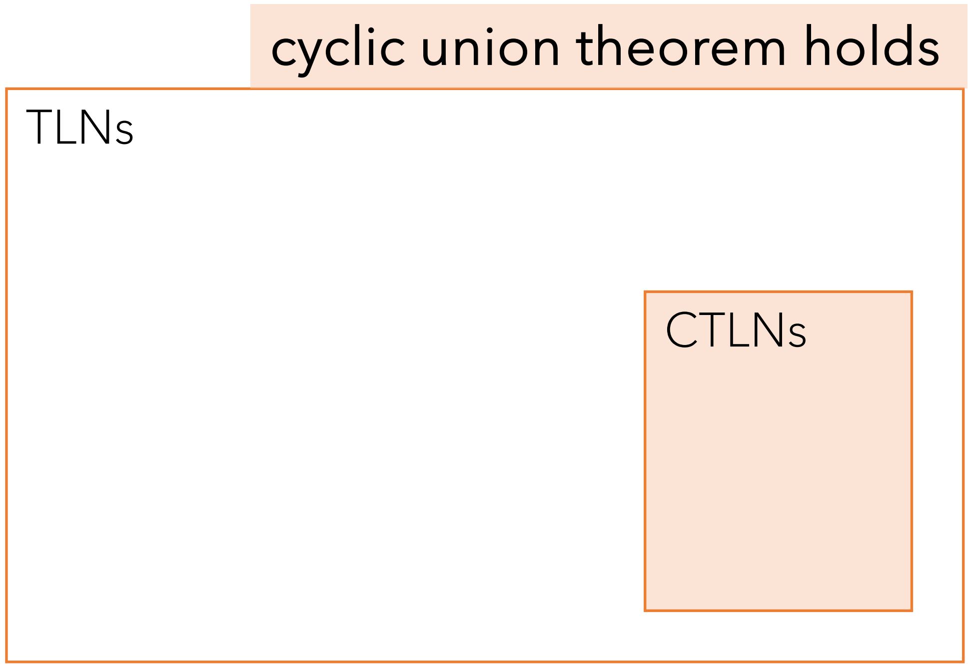


whose connectivity matrix is defined by a directed graph:



$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } j \to i \text{ in } G, \\ -1-\delta & \text{if } j \not\to i \text{ in } G, \end{cases}$$

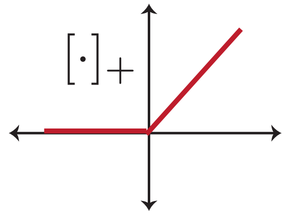
$$b_i = \theta$$



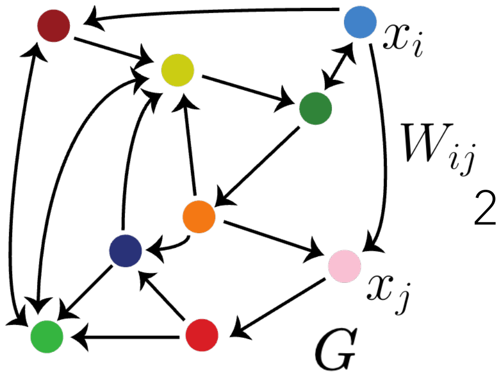
generalized combinatorial threshold-linear networks (gCTLNs)

threshold-linear networks:

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + b_i\right]_+ \qquad i = 1, \ldots, n$$



whose connectivity matrix is defined by a directed graph:



$$b_i = \theta$$

CTLN prescription:

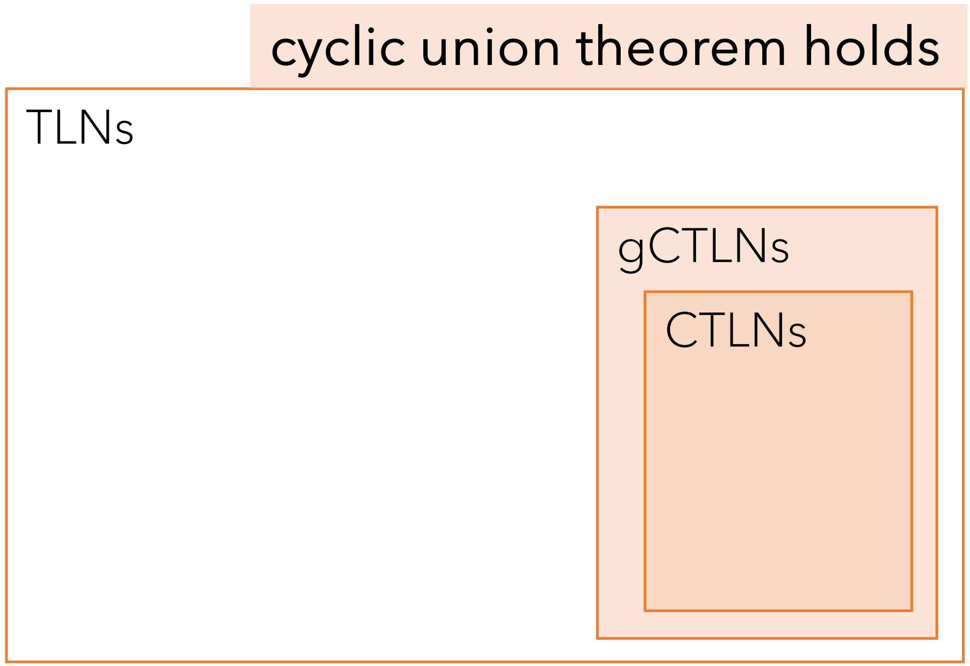
2 parameters

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1 + \varepsilon & \text{if } j \to i \text{ in } G, \\ -1 - \delta & \text{if } j \not\to i \text{ in } G, \end{cases}$$

gCTLN prescription:

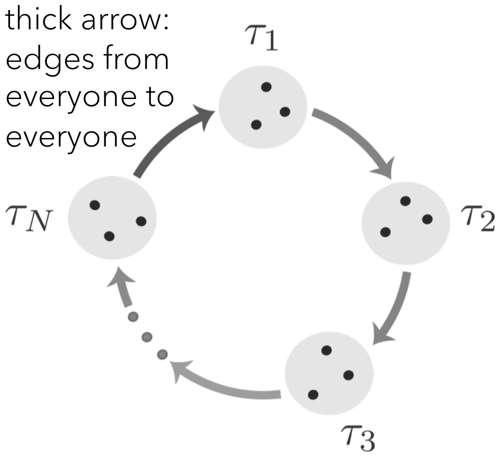
2n parameters

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1 + \varepsilon_j & \text{if } j \to i \text{ in } G, \\ -1 - \delta_j & \text{if } j \not\to i \text{ in } G, \end{cases}$$



cyclic union theorem holds for gCTLNs!

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$



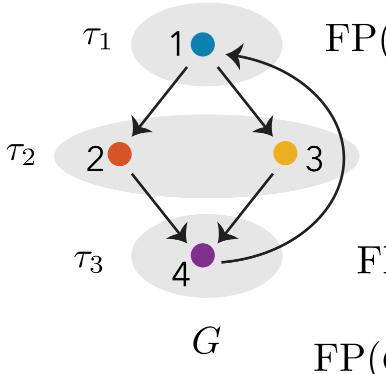
Cyclic union: edges forward from every node in previous component to every node in next component, and no other edges between components.

CTLN version:

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } j \to i \text{ in } G, \\ -1-\delta & \text{if } j \not\to i \text{ in } G, \end{cases}$$

Theorem: for all $i \in [N]$

$$\sigma \in \mathrm{FP}(G) \;\Leftrightarrow\; \sigma \cap \tau_i \in \mathrm{FP}(G|_{\tau_i})$$



$\mathrm{FP}(G|_{\tau_1}) = \{\{1\}\}$

$\mathrm{FP}(G|_{\tau_2}) = \{\{2\},\{3\},\{2,3\}\}$

$\mathrm{FP}(G|_{\tau_3}) = \{\{4\}\}$

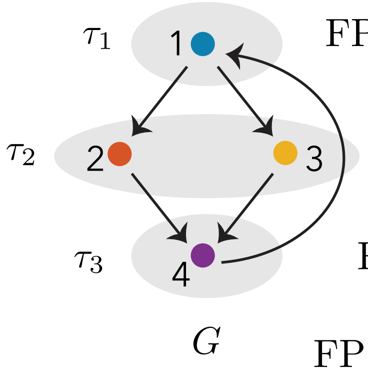
$\mathrm{FP}(G) = \{\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$

gCTLN version:

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon_j & \text{if } j \to i \text{ in } G, \\ -1-\delta_j & \text{if } j \not\to i \text{ in } G, \end{cases}$$

Theorem: for all $i \in [N]$

$$\sigma \in \mathrm{FP}(G,\varepsilon,\delta) \Leftrightarrow \sigma_i \in \mathrm{FP}(G|_{\tau_i},\varepsilon|_{\tau_i},\delta|_{\tau_i})$$



$\mathrm{FP}(G|_{\tau_1},\boldsymbol{\varepsilon}|_{\tau_1},\boldsymbol{\delta}|_{\tau_1}) = \{\{1\}\}$

$\mathrm{FP}(G|_{\tau_2},\boldsymbol{\varepsilon}|_{\tau_2},\boldsymbol{\delta}|_{\tau_2}) = \{\{2\},\{3\},\{2,3\}\}$

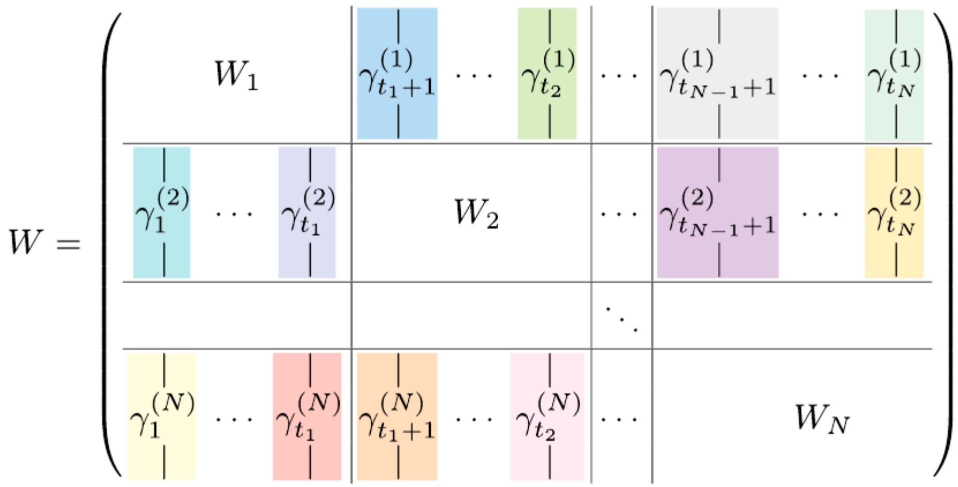
$\mathrm{FP}(G|_{\tau_3},\boldsymbol{\varepsilon}|_{\tau_3},\boldsymbol{\delta}|_{\tau_3}) = \{\{4\}\}$

$\mathrm{FP}(G,\boldsymbol{\varepsilon},\boldsymbol{\delta}) = \{\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$

The proof of the generalized cyclic union theorem for gCTLNs required several new ingredients (not so easy!)

1. menu theorem for TLNs (not graph-based!)

C simply-embedded partition



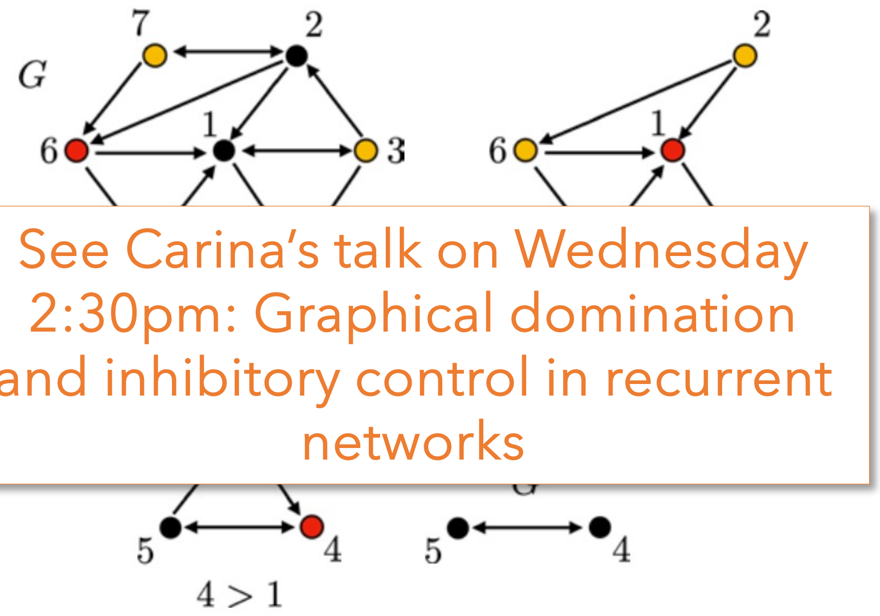
Theorem 3 (TLN menu). Let $\tau_1, \ldots, \tau_N$ be a simply-embedded partition of the nodes of (W, θ), with θ uniform input. For any $\sigma \subseteq [n]$, let $\sigma_\ell \stackrel{\text{def}}{=} \sigma \cap \tau_\ell$. Then

$$\sigma \in \mathrm{FP}(W, \theta) \quad \Rightarrow \quad \sigma_\ell \in \mathrm{FP}\left(W_{\tau_\ell}, \theta\right) \cup \{\emptyset\} \quad \text{for all } \ell \in [N].$$

In other words, every fixed point support of (W, θ) is a union of component fixed point supports σ_ℓ, at most one per component.

Londono Alvarez, 2025, *Fixed point decompositions for composite generalized combinatorial threshold-linear networks* (in preparation)

2. graphical domination results in gCTLNs



See Carina's talk on Wednesday 2:30pm: Graphical domination and inhibitory control in recurrent networks

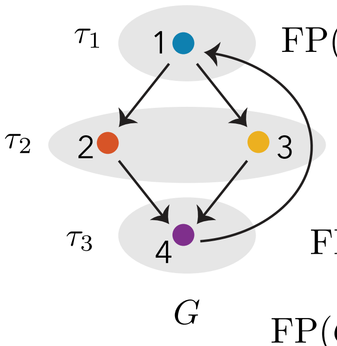
$$\mathrm{FP}(G) = \mathrm{FP}(\widetilde{G}) = \{45\}$$

Curto, 2025, *Notes on graphical domination and inhibitory control for threshold-linear networks with recurrent excitation and global inhibition* (in preparation)

the attractor heuristic seems to still work

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$

CTLN version:

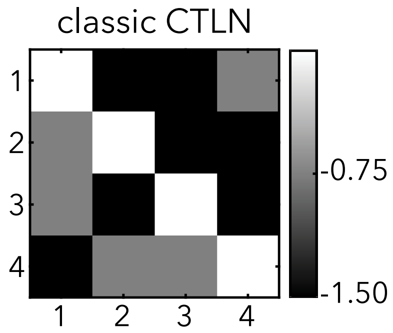


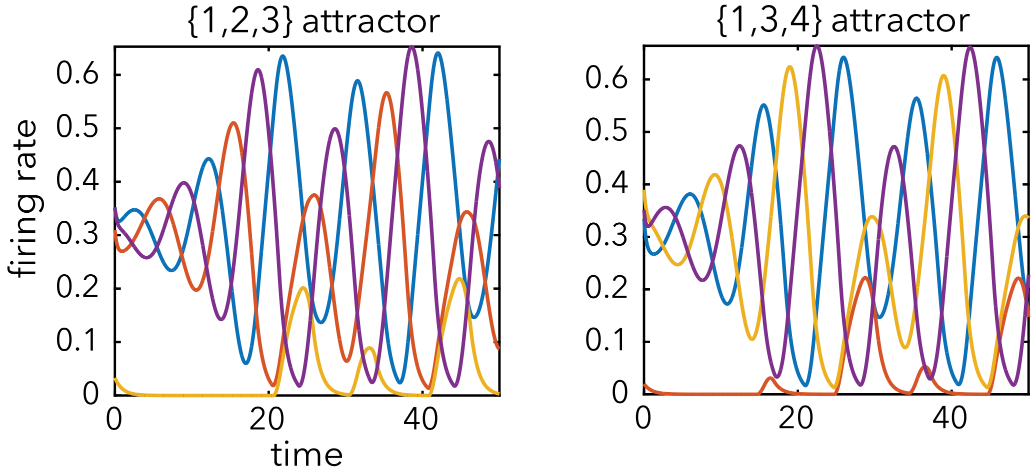
$\mathrm{FP}(G|_{\tau_1}) = \{\{1\}\}$

$\mathrm{FP}(G|_{\tau_2}) = \{\{2\},\{3\},\{2,3\}\}$

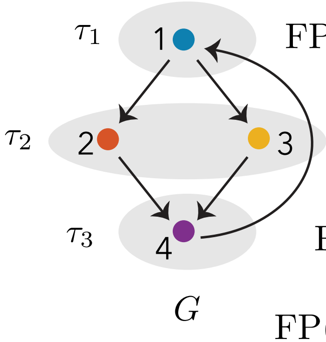
$\mathrm{FP}(G|_{\tau_3}) = \{\{4\}\}$

$\mathrm{FP}(G) = \{\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$





gCTLN version:

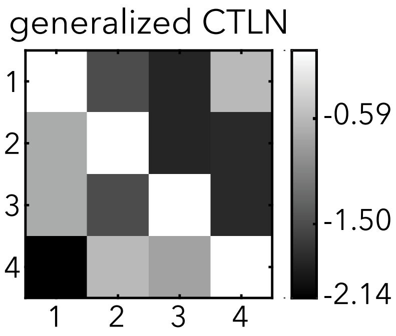


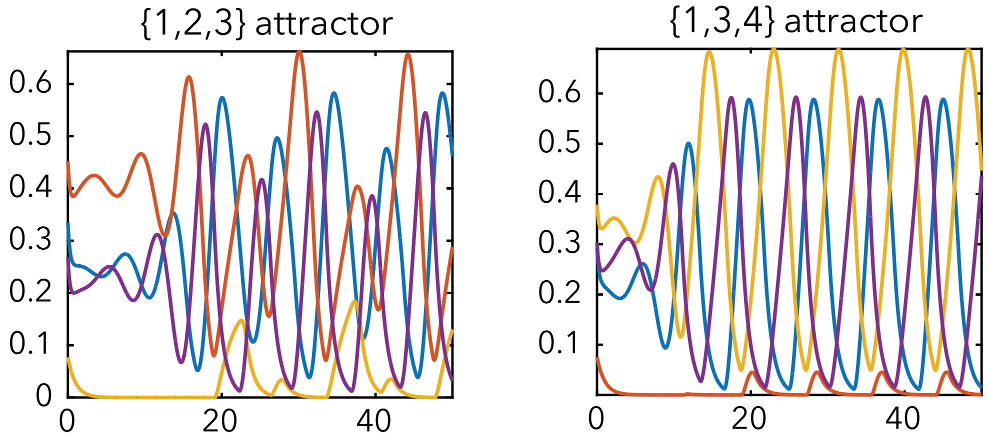
$\mathrm{FP}(G|_{\tau_1}, \boldsymbol{\varepsilon}|_{\tau_1}, \boldsymbol{\delta}|_{\tau_1}) = \{\{1\}\}$

$\mathrm{FP}(G|_{\tau_2}, \boldsymbol{\varepsilon}|_{\tau_2}, \boldsymbol{\delta}|_{\tau_2}) = \{\{2\},\{3\},\{2,3\}\}$

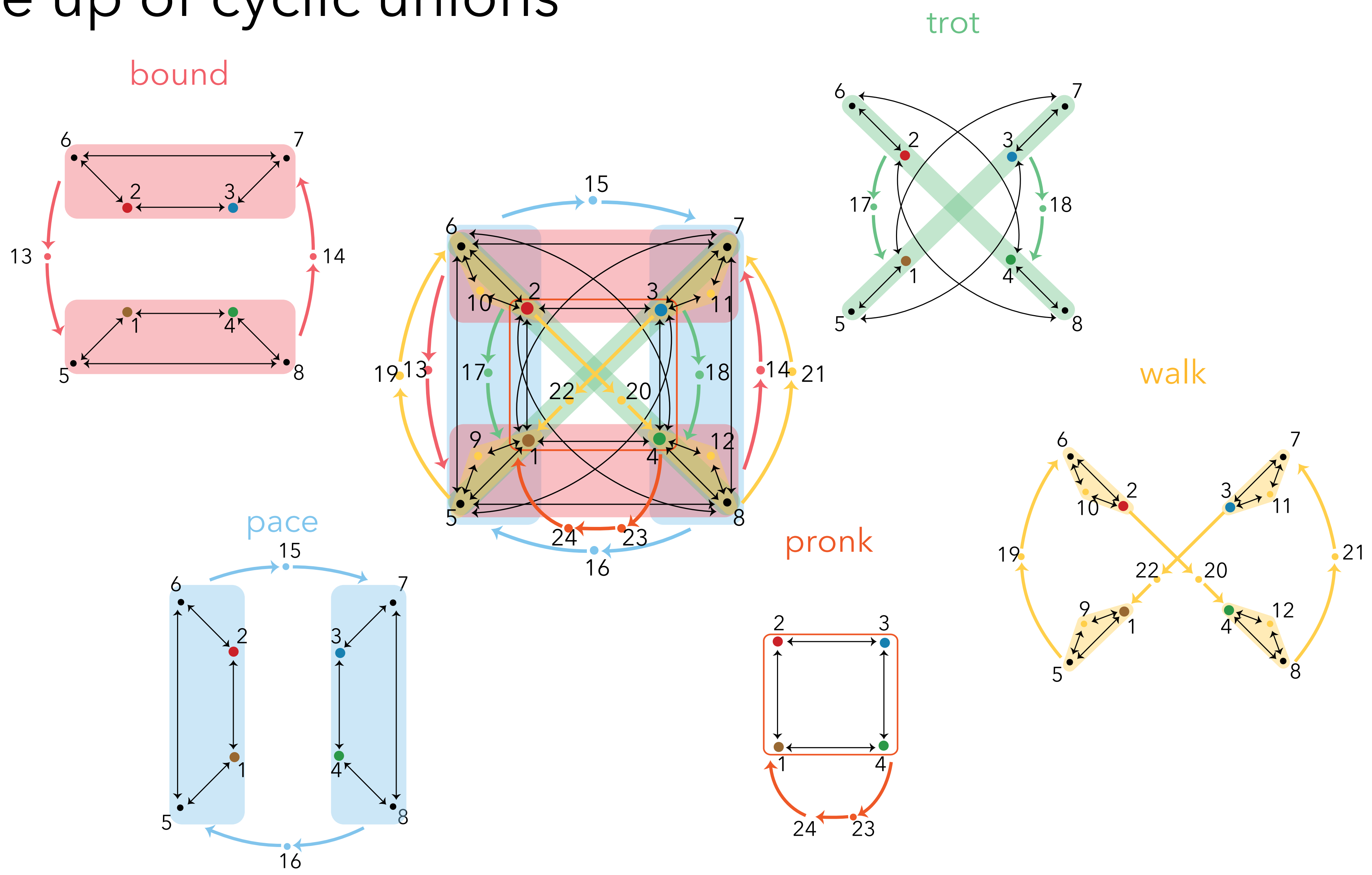
$\mathrm{FP}(G|_{\tau_3}, \boldsymbol{\varepsilon}|_{\tau_3}, \boldsymbol{\delta}|_{\tau_3}) = \{\{4\}\}$

$\mathrm{FP}(G, \boldsymbol{\varepsilon}, \boldsymbol{\delta}) = \{\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$

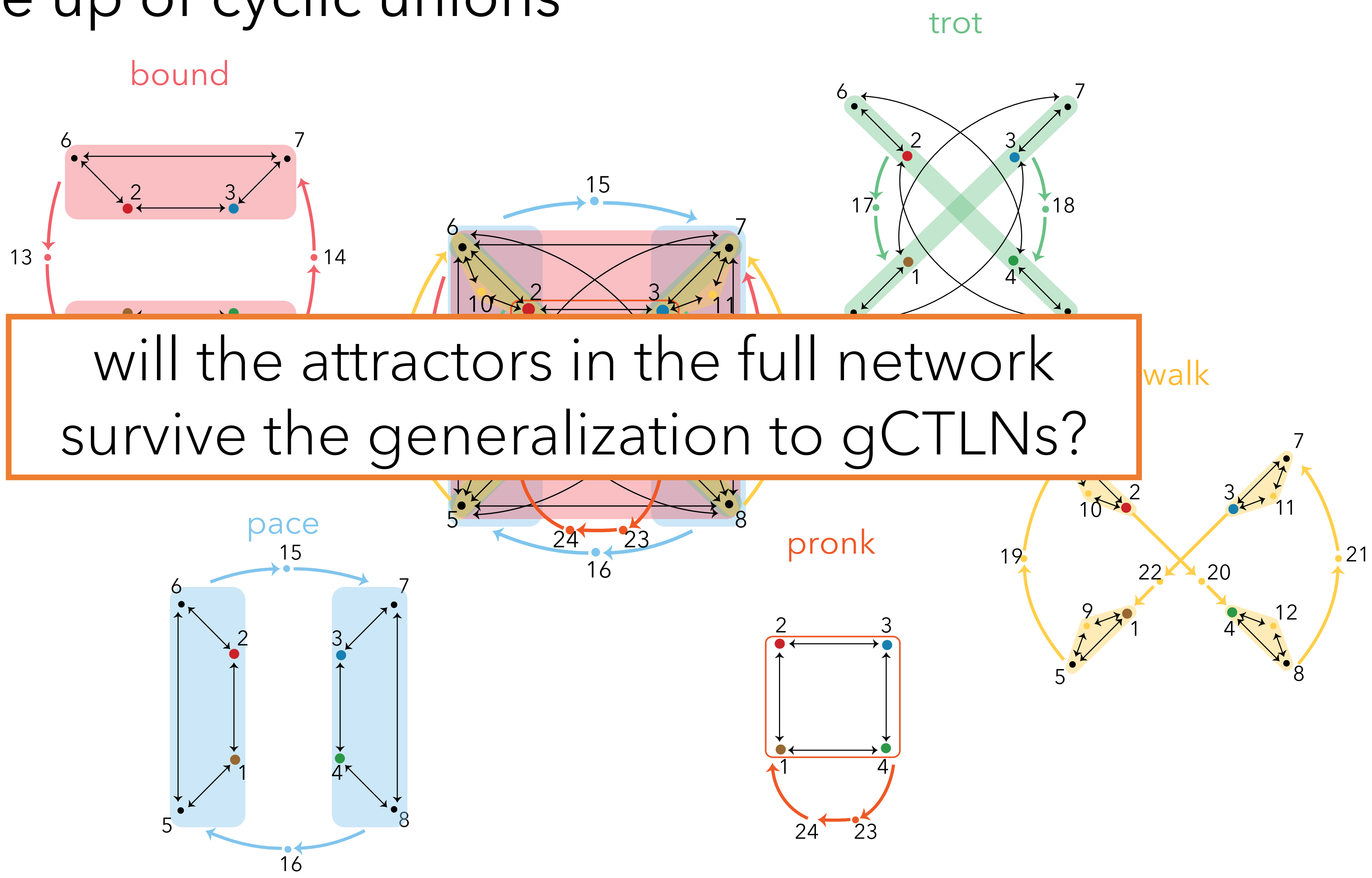




while full network was not technically not a cyclic union, it was made up of cyclic unions

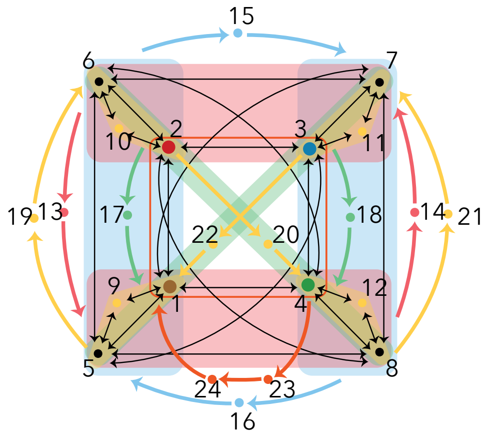


while full network was not technically not a cyclic union, it was made up of cyclic unions



will the attractors in the full network survive the generalization to gCTLNs?

the answer is: sometimes

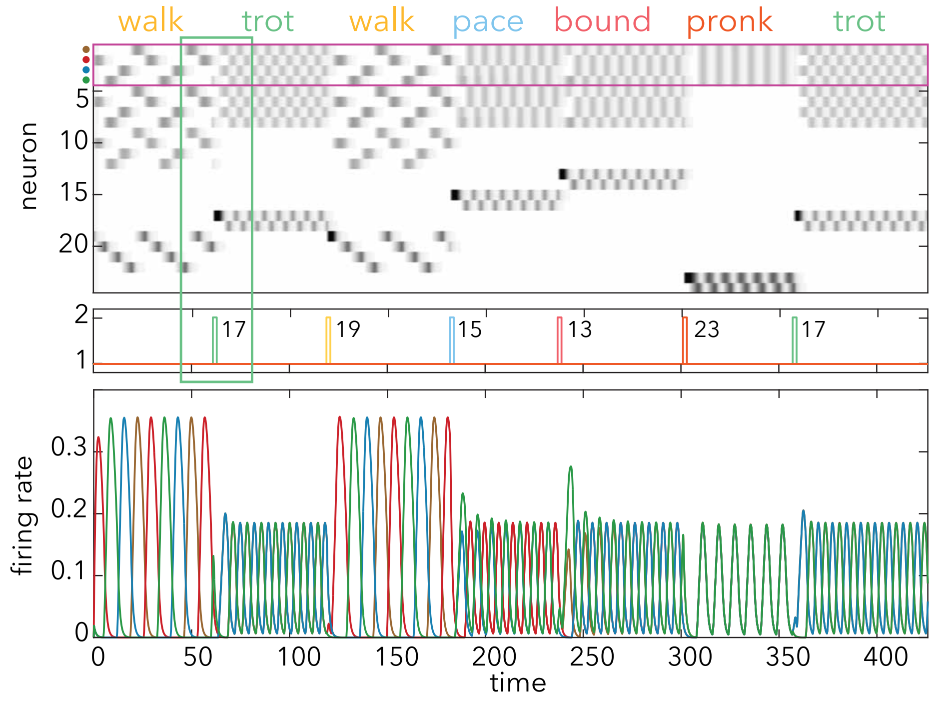


CTLN prescription:

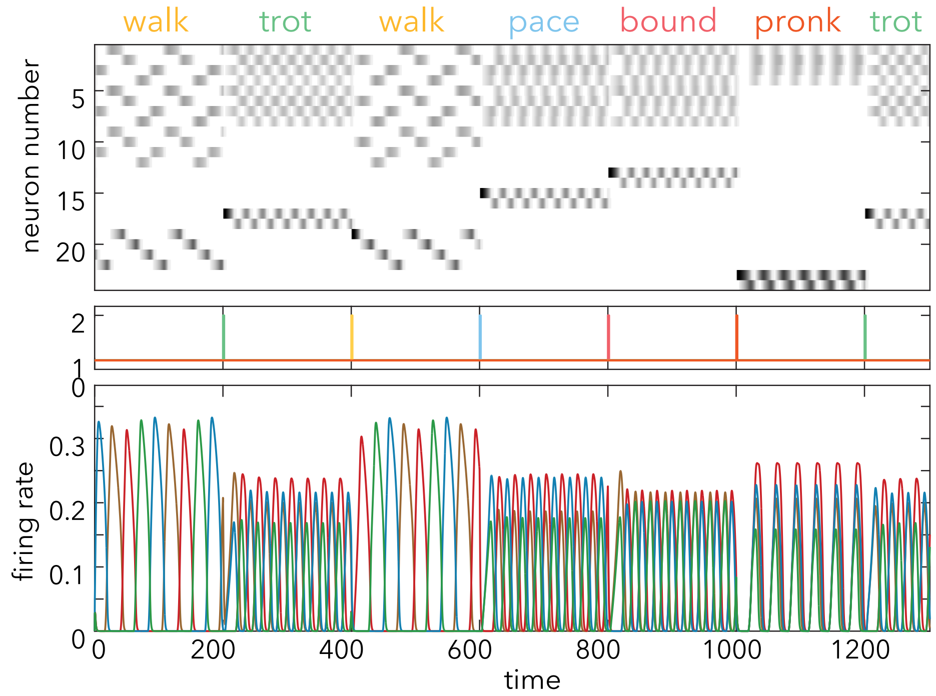
$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon & \text{if } j \to i \text{ in } G, \\ -1-\delta & \text{if } j \not\to i \text{ in } G, \end{cases}$$

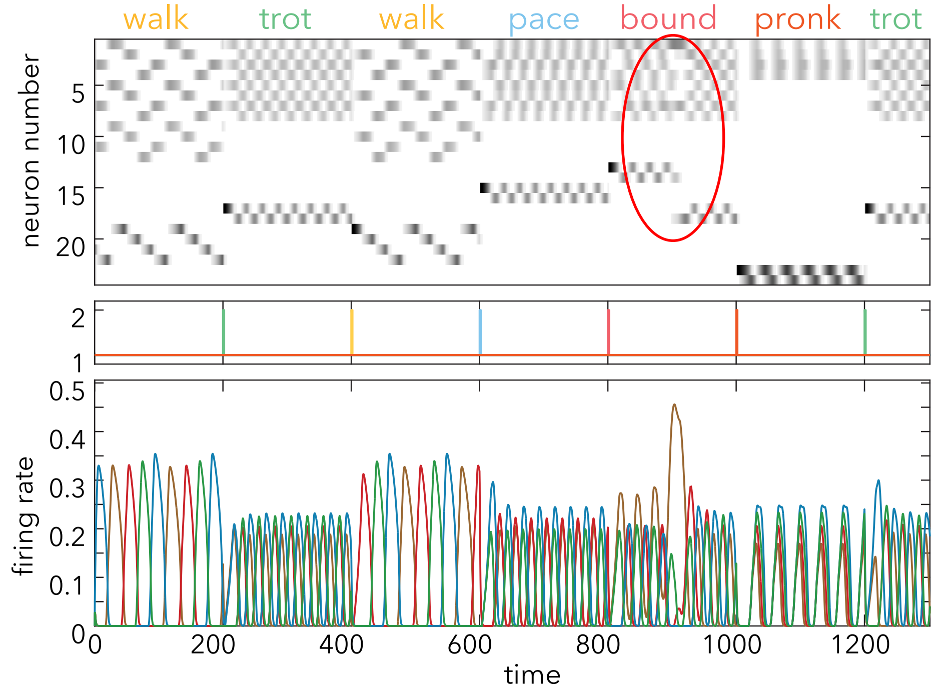
gCTLN prescription:

$$W_{ij} = \begin{cases} 0 & \text{if } i = j, \\ -1+\varepsilon_j & \text{if } j \to i \text{ in } G, \\ -1-\delta_j & \text{if } j \not\to i \text{ in } G, \end{cases}$$



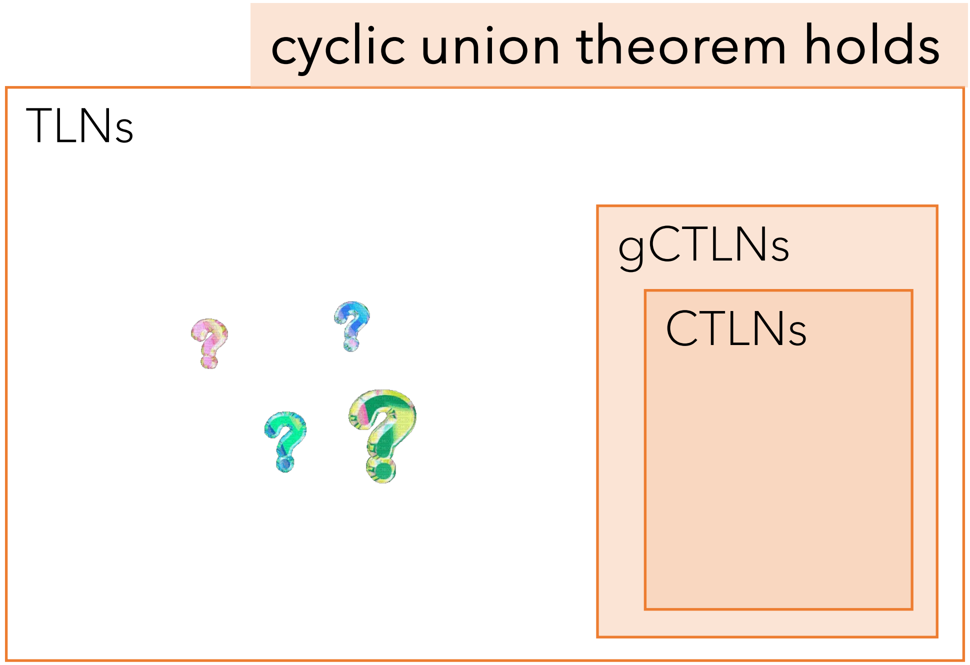
two gCTLNs amounting to the same level of perturbation from original parameters:



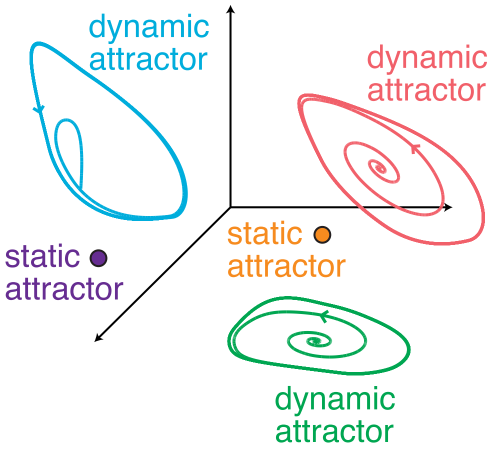


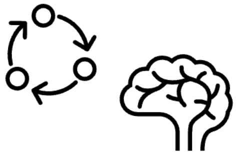
Looking ahead:

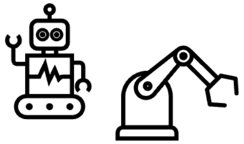
- how much can we generalize these fixed point decomposition results?
- what controls the firing rates? bifurcations?
- how to “glue” attractors (as in quadruped gaits)?



$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$



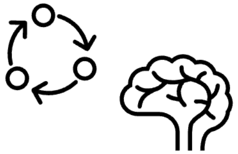


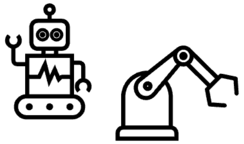


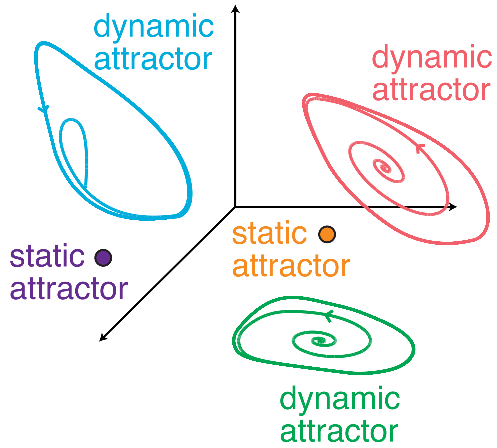
In conclusion, TLNs are a nice framework for understanding structure-function relationships, and have potential as biological and artificial models

Thank you!

$$\frac{dx_i}{dt} = -x_i + \left[\sum_{j=1}^{n} W_{ij}x_j + \theta\right]_+$$







In conclusion, TLNs are a nice framework for understanding structure-function relationships, and have potential as biological and artificial models

Sequential Attractors in Combinatorial Threshold-Linear Networks

CTLN architectures giving rise to sequential attractors:
Parmelee, C., **Londono Alvarez, J.**, Curto, C., & Morrison, K. (2022). Sequential attractors in combinatorial threshold-linear networks. *SIAM Journal on Applied Dynamical Systems*, 21(2), 1597-1630.

quadruped gaits preprint

Quadruped locomotion and more:
Londono Alvarez, J., Morrison, K., & Curto, C. (2025). *Attractor-based models for sequences and pattern generation in neural circuits*. bioRxiv.